Gravitational Redshift

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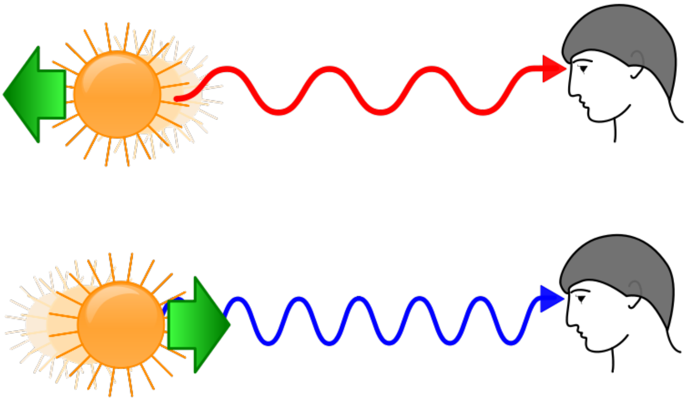
Review

- In lecture 29, we found that photons get delayed when climbing out of a gravitational well
- In this lecture, we focus on another curious property of photons climbing out of a gravitational well
- We will show that photons are *redshifted*, that is, their frequency (and energy) is lowered when climbing out of a gravitational well

The Doppler Effect

Special Relativistic Doppler Effect



Redshift in GR

- There is a Doppler Effect in Special Relativity for moving objects
- Objects moving away from an observer appear red-shifted
- There is an *additional Doppler effect* in GR
- Similar to what we found in lecture 29, the Doppler effect is present even for non-moving objects

Doppler effect in GR

- Let's consider two observers, located at coordinates $r = r_1$ and $r = r_2$ in a Schwarzschild spacetime
- The observers are not moving, so $\frac{d\vec{x}}{dt} = 0$ for both
- Observers one and two have proper times τ_1, τ_2 given by

$$d\tau_1^2 = -g_{\mu\nu} dx^\mu dx^\nu\big|_{r=r_1} = A(r_1) dt^2\,, \tag{30.1}$$

$$d\tau_2^2 = -g_{\mu\nu} dx^\mu dx^\nu\big|_{r=r_2} = A(r_2) dt^2\,. \tag{30.2}$$

- Since we use *the same global time coordinate t* for both observers, we have

$$dt^2 = \frac{d\tau_1^2}{A(r_1)} = \frac{d\tau_2^2}{A(r_2)}\,. \tag{30.3}$$

Doppler effect in GR

- The local proper time-intervals for observers one and two are related by

$$\Delta\tau_1 = \sqrt{\frac{1-\frac{2GM}{r_1}}{1-\frac{2GM}{r_2}}}\Delta\tau_2\,. \tag{30.4}$$

- The local frequencies are inverse time intervals $\omega \propto \frac{1}{\Delta t}$
- We find that frequencies measured by observer one and two are related by

$$\omega_1 = \sqrt{\frac{1-\frac{2GM}{r_2}}{1-\frac{2GM}{r_1}}}\omega_2\,. \tag{30.5}$$

- The frequencies differ even though the observers are not moving!

Doppler effect in GR

- Let's consider observer two to be very far away from the central mass:

$$r_2 \to \infty \tag{30.6}$$

- Denoting the frequency measured by observer two as $\omega_2 \to \omega_\infty$, we have

$$\omega_\infty = \omega \times \sqrt{1 - \frac{2GM}{r}} \tag{30.7}$$

where r, ω are the radius and frequency of observer one
- Since $\sqrt{1 - \frac{2GM}{r}} < 1$ for $r > r_s$, $\omega_\infty < \omega$
- The frequency ω appears *redshifted* to the distant observer

Gravitational Redshift

- Since both the emitter and observer are at rest, the redshift is not a result of movement
- Instead, it is a *gravitational effect*
- There is a simple physical interpretation:
- The frequency of a photon is proportional to its energy,

$$E = 2\pi\omega \tag{30.8}$$

- However, deep in the gravitational well, the photon has little potential energy
- In order to climb out of the gravitational well, the photon needs to overcome the gravitational potential $\frac{GM}{r}$

Gravitational Redshift – Newton Theory

- Let's estimate the gravitational redshift in Newton's theory of gravity
- A photon of frequency ω at position r in a gravitational well of mass M has potential energy

$$E_{\rm pot} = -\frac{GM}{r} \times 2\pi\omega\,. \tag{30.9}$$

- It's total energy therefore is

$$E_{\rm tot} = 2\pi\omega\left(1 - \frac{GM}{r}\right)\,. \tag{30.10}$$

- After leaving the potential well at $r \to \infty$, the photon's energy is

$$E_{\rm tot} = 2\pi\omega_\infty = 2\pi\omega\left(1 - \frac{GM}{r}\right) \tag{30.11}$$

Newton vs. GR

- In Newton gravity, we therefore expect

$$\omega_\infty = \omega \times \left(1 - \frac{GM}{r}\right) \tag{30.12}$$

- By contrast, in GR we had (30.7)

$$\omega_\infty = \omega \times \sqrt{1 - \frac{2GM}{r}} \simeq \omega \times \left(1 - \frac{GM}{r} + \ldots\right) . \tag{30.13}$$

- The GR calculation matches Newton gravity for *weak fields* $\frac{GM}{r} \ll 1$

Extreme Gravitational Redshift

- For ordinary stars, the star's radius $r \gg 2GM = r_s$
- For ordinary stars, the gravitational redshift

$$\omega_\infty = \omega \times \sqrt{1 - \frac{r_s}{r}} \tag{30.14}$$

is never extreme

- However, what if a photon tries to climb out of a gravitational well from radius $r = r_s$?
- As $r \to r_s + 0^+$, we have

$$\lim_{r \to r_s} \omega_\infty \to 0\,. \tag{30.15}$$

- The redshift is extreme, the apparent color is black