Hawking Radiation

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Review

- In lecture 31, we discussed Black Holes
- It was mentioned that black holes are not entirely black, they emit something known as Hawking radiation
- Hawking radiation is an effect that arise from combining quantum mechanics with gravity – this is DIFFERENT from quantum gravity
- We will calculate some properties of Hawking radiation in the present lecture

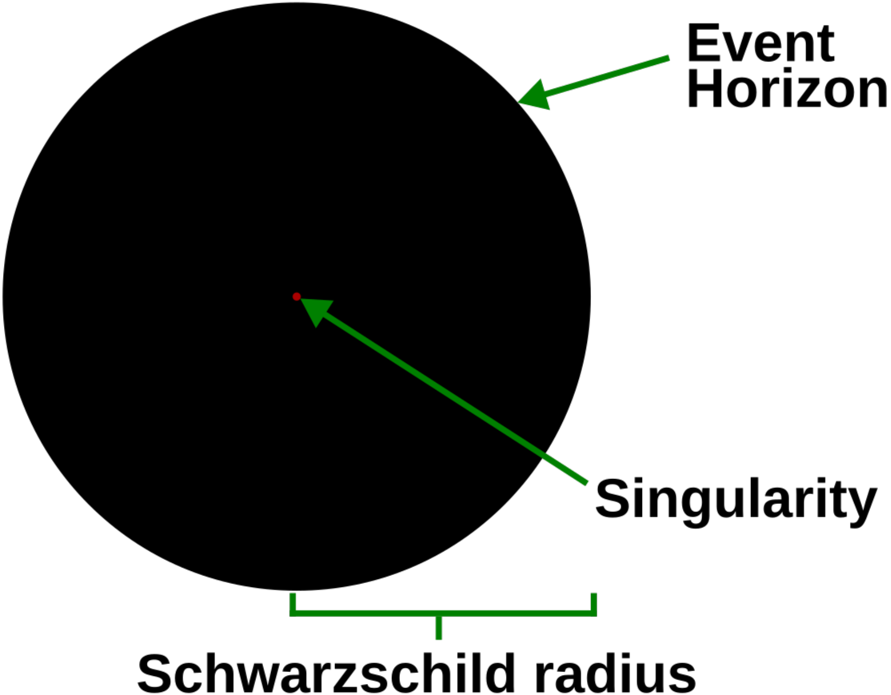
Review: Schwarzschild solution

- Let's recall the properties of the Schwarzschild solution to GR
- It's a static and spherically symmetric solution to Einstein's field equations
- In spherical coordinates:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2 \tag{32.1}$$

  where $A(r) = 1 - \frac{r_s}{r}$
- The solution has a *coordinate singularity* at the horizon

$$r = r_s = 2GM\,. \tag{32.2}$$


Event
Horizon
Singularity
Schwarzschild radius

Hawking Radiation – Physics Intuition

- Before jumping into the math, let's consider physics first
- You know that in quantum field theory, we have a process known as pair creation
- This process allows a particle-antiparticle pair to appear out of the vacuum for a brief moment of time
- After this short time, the pair annihilates, giving its energy back to the vacuum
- Think of it as a zero-interest mortgage
- If such a process happens very close to the horizon of a black hole, it may happen that during the time it exists, the anti-particle falls into the black hole, while the other is still outside
- If the particle escapes, then it's energy must have come from somewhere – namely the black hole

Hawking Radiation – Physics Intuition

- To re-iterate: in QFT particles can "borrow" energy from the vacuum
- Normally, they give their energy back to the vacuum by annihilating
- However, if their annihilation partner falls into the BH, they cannot annihilate
- So instead, the black hole has to supply the energy for the remaining particle outside
- Since the black hole has to give up energy, it looses mass

Hawking Radiation – Math Derivation

- Let's now derive some properties of Hawking radiation using math
- Recall that the Schwarzschild metric (32.1) has a coordinate singularity at the horizon $r = r_s$
- Let's focus on the region close to the horizon by changing coordinates:

$$r = r_s(1+\rho^2)\,, \quad \rho \ll 1\,. \tag{32.3}$$

- For the near-horizon region, the metric (32.1) becomes

$$ds^2 = -(1-1+\rho^2)dt^2 + \frac{r_s^2 4\rho^2 d\rho^2}{\rho^2} + r_s^2 d\Omega^2\,, \tag{32.4}$$

$$= -\rho^2 dt^2 + 4 r_s^2 d\rho^2 + r_s^2 d\Omega^2\,. \tag{32.5}$$

Hawking Radiation – Temperature

- We find

$$ds^2 = 4r_s^2\left(-\frac{\rho^2}{4r_s^2}dt^2 + d\rho^2 + \frac{1}{4}d\Omega^2\right) . \tag{32.6}$$

- Let's now go to *imaginary time* as in thermal quantum field theory:

$$t = i\tau\,, \quad \tau \in [0, \frac{1}{T}] \tag{32.7}$$

where T is the temperature of the system

- This gives

$$ds^2 = 4r_s^2\left(\frac{\rho^2}{4r_s^2}d\tau^2 + d\rho^2 + \frac{1}{4}d\Omega^2\right) . \tag{32.8}$$

Hawking Radiation – Temperature

- We get a piece of the line element that looks like

$$ds^2 = \frac{\rho^2}{4r_s^2}d\tau^2 + d\rho^2 \tag{32.9}$$

- That is very similar to the line-element in polar coordinates

$$ds^2 = \rho^2 d\phi^2 + d\rho^2 \tag{32.10}$$

- So we can identify an "angle" $\phi = \frac{\tau}{2r_s}$ in (32.8) such that
- The angle ϕ needs to be period with period 2π or else the space-time will have a conical singularity

Hawking Radiation – Temperature

- We find that in order to avoid a conical singularity in the space-time, we must have periodicity

$$\frac{\tau}{2r_s} = \frac{\tau}{2r_s} + 2\pi\,. \tag{32.11}$$

- However, we also need τ to be period with inverse temperature:

$$\tau = \tau + \frac{1}{T}\,. \tag{32.12}$$

- Since these two conditions must be the same, we find that $\frac{1}{T} = 4\pi r_s$ or

$$T = \frac{1}{4\pi r_s} = \frac{1}{8\pi GM} = T_{\rm Hawking}\,. \tag{32.13}$$

Hawking Radiation – Interpretation

- We find that Black Holes radiate
- The radiation spectrum is that of a perfect black body with temperature

$$T = T_{\rm Hawking} = \frac{1}{8\pi GM}\,. \tag{32.14}$$

- The bigger the BH's mass, the *smaller* the temperature
- Small black holes are hotter than large black holes
- This means black holes eventually evaporate!