The Age of The Universe

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Review

- In lecture 35, we found the Friedmann equations for cosmology
- Using the shorthand notation $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ these are

$$\partial_t \epsilon + 3(\epsilon + p)H = 0 \,, \quad 3\left(H^2 + \frac{\kappa}{a^2}\right) = 8\pi G \epsilon \,. \tag{36.1}$$

- These equations describe the dynamics of a spacetime with FLRW metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right) \,, \tag{36.2}$$

  where $\kappa = -1, 0, 1$
- In this lecture, we focus on solutions to (36.1) describing an expanding universe

Scale Factor

- Let's first take another look at the FLRW metric (36.2)
- For flat space $\kappa = 0$, the distance between two points with $dt = 0$ is given by
$$(\Delta x)^2 = a(t)^2 \left(dr^2 + r^2 d\Omega^2\right) . \tag{36.3}$$
- For a line at fixed angle $d\Omega = 0$, so
$$\Delta x = a(t)\Delta r \tag{36.4}$$
- This means the distance Δx between two points is proportional to $a(t)$
- We call $a(t)$ the **scale factor** of the universe

Hubble constant

- Let's Taylor expand $a(t)$ around some time $t \simeq t_0$
- One gets

$$a(t) = a(t - t_0 + t_0) = a(t_0) + (t - t_0)\dot{a}(t_0) + \frac{(t-t_0)^2}{2}\ddot{a}(t_0) + \ldots \tag{36.5}$$

- Dividing by $a(t_0) \neq 0$ and re-arranging gives

$$\frac{a(t)}{a(t_0)} = 1 - (t_0 - t)\frac{\dot{a}(t_0)}{a(t_0)} - \frac{(t_0 - t)^2}{2}\left(-\frac{\ddot{a}(t_0)}{a(t_0)}\right) + \ldots \tag{36.6}$$

- We defined $H(t) = \frac{\dot{a}(t)}{a(t)}$ so $H_0 \equiv \frac{\dot{a}(t_0)}{a(t_0)}$ is the **Hubble constant**

Deceleration parameter

- Let's further define
$$q_0 \equiv -\frac{1}{H_0^2}\frac{\ddot{a}(t_0)}{a(t_0)} \tag{36.7}$$
- We will call q_0 the **deceleration parameter**

Expanding Universe

- In terms of these, we have

$$\frac{a(t)}{a(t_0)} = 1 - (t_0 - t)H_0 - \frac{(t_0 - t)^2}{2} q_0 H_0^2 + \ldots \qquad (36.8)$$

- If t_0 is **today** and $t < t_0$ is in the past then

$$\frac{a(t)}{a(t_0)} = 1 - (t_0 - t)H_0 - \frac{(t_0 - t)^2}{2} q_0 H_0^2 \qquad (36.9)$$

The Age of The Universe

- If we approximate by linear term

$$\frac{a(t)}{a(t_0)} \simeq 1 - (t_0 - t)H_0 \tag{36.10}$$

- At the *beginning of the universe* $t = 0$
- At the *beginning of the universe* $a(0) = 0$, so

$$0 = 1 - t_0 H_0 \tag{36.11}$$

- This gives us **the age of the universe** as

$$t_0 = \frac{1}{H_0}\,. \tag{36.12}$$

- It is also known as "Hubble time"

Hubble Constant

- From observations

$$H_0 = 100 \frac{km/s}{Mpc} \times h \tag{36.13}$$

  where historically $h = 1$ (now: $h \simeq 0.7$)

- This implies for the age of the universe

$$t_0 \simeq \frac{10^{-2}}{h} \frac{Mpc}{km/s} = \frac{9.78}{h} \mathrm{Gyr}\,. \tag{36.14}$$

- Problem: metal-poor stars in the Milky Way have age $\sim 15.2 \pm 3.7$ Gyr
- For $h = 1$ this was known as the **age problem**