Our Universe

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Review

- In lecture 37, we derived a balance equation for the geometry of the universe
- Using the notation $\Omega \equiv \frac{8\pi G\epsilon}{3H^2}$ we found

$$\frac{\kappa}{a^2 H^2} = \Omega - 1\,. \tag{38.1}$$

- For an equation of state $p = w\epsilon$, we found for the energy density

$$\epsilon(t) = \epsilon_0 \left(\frac{a_0}{a}\right)^{3(1+w)} \tag{38.2}$$

- In lecture 37, we found a solution for $a(t)$ assuming $\kappa = 0$ (flat universe)
- In this lecture, we will explore what happens for $\kappa \neq 0$

Review – Geometry

- Recall that the value of κ corresponds to the overall geometry of space
- For $\kappa = 0$, we have flat space, $R = 0$ and $\Lambda = 0$
- For $\kappa = 1$, we have elliptic space, $R > 0$ and $\Lambda > 0$
- For $\kappa = -1$, we have hyperbolic space, $R < 0$ and $\Lambda < 0$

Geometry of the Universe

- We considered the case of $\kappa = 0$ (flat universe) in lecture 37
- From (38.1) we have $\kappa = 0 \leftrightarrow \Omega = 1$
- Now for $\kappa < 0$, (38.1) implies $\Omega < 1$; we have an **under-dense** universe
- By contrast for $\kappa > 0$, (38.1) implies $\Omega > 1$: we have an **over-dense** universe

Energy Density and Geometry

- Depending on value of Ω, we have a different geometry of the universe
- Knowing Ω is important!
- The value of Ω is given by the sum over all matter contributions:

$$\Omega = \Omega_M + \Omega_R + \Omega_\Lambda \tag{38.3}$$

- Each of these contributions comes with a different EoS parameter:

$$\text{Matter}: w = 0\,, \quad \text{Radiation}: w = \frac{1}{3}\,, \quad \text{CC}: w = -1\,. \tag{38.4}$$

- As a consequence, the energy-densities (38.2) scale differently:

$$\epsilon_M \propto a^{-3}\,, \quad \epsilon_R \propto a^{-4}\,, \quad \epsilon_\Lambda \propto a^{0}\,. \tag{38.5}$$

- In addition, lhs of (38.1) scales as $\epsilon_\kappa \propto a^{-2}$; **Complicated scale factor evolution!**

Our Universe

We have observational data for our universe:

- $\Omega \simeq 1 = \Omega_M + \Omega_R + \Omega_\Lambda$; **our universe is almost flat!**
- $\Omega_{M,0} \simeq 0.3 \pm 0.1$; **about 1/3 is matter**
- $\Omega_{R,0} \simeq 0$; **almost no radiation**
- As a consequence, we must have $\Omega_{\Lambda,0} \simeq 0.7$; **the lion's share of Ω is vacuum energy!**

More Details: Radiation

- Today, most radiation in the universe is from the Cosmic Microwave Background (CMB)
- This radiation permeates the universe, and it's almost black-body with $T \simeq 2.7$ K
- The energy density for radiation is

$$\epsilon_R = \frac{\pi^2 T^4}{90} \times 2 \tag{38.6}$$

- The critical energy density today is

$$\epsilon_c = \frac{3H_0^2}{8\pi G} \simeq 5 GeV m^{-3} \,. \tag{38.7}$$

- Using unit conversion, this gives

$$\Omega_{R0} = \frac{\epsilon_R}{\epsilon_c} \simeq 2 \times 10^{-5} \tag{38.8}$$

More Details: Matter

- Observational data suggests that

$$\Omega_{M0} \simeq 0.3 \tag{38.9}$$

- **Normal** matter is mostly in stars; for stars, we can calculate

$$\Omega_{\rm stars} \simeq 0.046\,. \tag{38.10}$$

- This leaves a gap of

$$\Omega_{M0} - \Omega_{\rm stars} \simeq 0.25\,. \tag{38.11}$$

- **We have no idea what this matter is!**
- We call it "Dark Matter"

More Details: Dark Energy

- From the energy budget (38.3), we found

$$\Omega_{\Lambda 0} \simeq 0.7 \tag{38.12}$$

- **We cannot calculate Ω_Λ!**
- A naive estimate gives $\Omega_{\Lambda 0} \simeq 10^{120}$, which is quite a bit off
- We call Ω_Λ **dark energy** (mostly because it sounds mysterious)
- We **could** calculate Ω_Λ if we understood how to consistently formulate a **quantum theory of gravity**

Coincidence problem

- Observational data suggests

$$\Omega_{R0} \propto 10^{-5}\,, \quad \Omega_{M0} \propto 0.3\,, \quad \Omega_{\Lambda 0} \propto 0.7\,. \tag{38.13}$$

- We know that

$$\Omega_R \propto a^{-4}\,, \quad \Omega_M \propto a^{-3}\,, \quad \Omega_\Lambda \propto a^0\,. \tag{38.14}$$

- Over the history of the universe, $a(t)$ changes dramatically
- Why is $\Omega_M \propto \Omega_\Lambda$ **today**?
- This is known as the **coincidence problem**