Inflation

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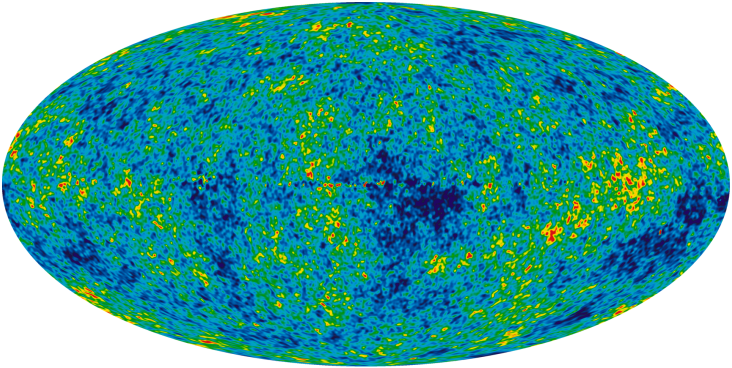
Review

- We have studied the evolution of the scale factor in the FLRW universe
- The solutions depend on the matter content of the universe
- By "matter" we mean: matter (incl. "dark" matter), radiation, dark energy
- So far, we have focused on solutions dominated by matter and radiation
- In this lecture, we want to look at dark energy

Horizon Problem



Review – Cosmology

- Recall from the lecture on Friedmann Equations (37) that the solution for the energy density is given by

$$\epsilon(t) = \epsilon_0 \left(\frac{a_0}{a(t)}\right)^{3(1+w)} \tag{40.1}$$

- To find a solution for the scale factor, we also have to solve

$$H^2 = \frac{8\pi G}{3}\epsilon - \frac{\kappa}{a^2} \tag{40.2}$$

- Let's now look for solutions assuming *pure dark energy*

Pure Dark Energy

- Let's assume that there is no matter and no radiation in the universe
- All there is is a cosmological constant Λ ("dark energy")
- Pure dark energy has an equation of state $p = -\epsilon$, so $w = -1$
- From (40.1) this gives

$$\epsilon(t) = \epsilon_0 = \text{const.} \tag{40.3}$$

- The evolution equation for the scale factor thus is

$$H^2 = \frac{8\pi G}{3}\epsilon_0 - \frac{\kappa}{a^2}\,. \tag{40.4}$$

Solving for the Scale Factor

- Using the definition of $H = \frac{\dot{a}}{a}$, the evolution equation for the scale factor becomes
$$\dot{a}^2 = a^2 \frac{8\pi G}{3} \epsilon_0 - \kappa \,. \tag{40.5}$$
- If $a \gg 1$, then the first term dominates over κ
- We get
$$\dot{a} = \pm a \sqrt{\frac{8\pi G \epsilon_0}{3}} \,. \tag{40.6}$$
- Assuming a *growing universe* (with $\dot{a} > 0$), we find the solution
$$a(t) \propto e^{t\sqrt{\frac{8\pi G\epsilon_0}{3}}} \tag{40.7}$$

Solving for the Scale Factor

- We find an exponentially increasing scale factor!
- This is known as **inflation**
- The universe expands so rapidly that even light cannot compete — everything gets smoothed out
- This inflationary period solves two problems in cosmology: the flatness problem ($\Omega_0 \simeq 1$) and the horizon problem (CMB)
- For this reason, inflation is considered a key component in standard cosmology!

Problems with Inflation

- There are also a number of problems with inflation
- One of the problems is that we cannot calculate Λ
- There are additional problems
- First consider $\Lambda > 0$. In this case $\epsilon_0 > 0$ and $\kappa = 1$, so we have

$$\dot{a}^2 = \frac{8\pi G \epsilon_0}{3} a^2 - 1 \,. \tag{40.8}$$

- For large a, everything is fine, but a was much smaller at earlier times
- We find that in order for (40.8) to make sense, there is a *minimum size* of the universe given by

$$a_{\rm min} = \sqrt{\frac{8\pi G |\epsilon_0|}{3}} \tag{40.9}$$

Problems with Inflation

- Observations suggest $\kappa = -1$
- This would imply $\Lambda < 0$. In this case $\epsilon_0 < 0$ so we have

$$\dot{a}^2 = -\frac{8\pi G|\epsilon_0|}{3} a^2 + 1\,. \tag{40.10}$$

- In this scenario, there is no inflation at all because $a < a_{\rm min}$!