AdS/CFT

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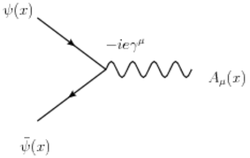
Review

- So far, we have focused on applications of GR to Astrophysics and Cosmology
- However, there is another application of GR: quantum field theory
- This is because there is a (conjectured) duality between GR and QFT originating from string theory – the AdS/CFT conjecture
- This duality is very powerful, and uses GR methods to solve hard problems in QFT
- In this lecture, some aspects of AdS/CFT will be highlighted

Point Particles

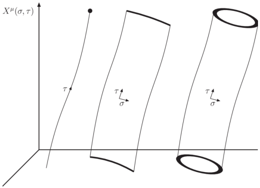
- In Quantum Field Theory, fundamental particles are point-like
- If particles move, their trajectory is a curve (called **worldline**)

- If particles interact, their interaction is in the form of a vertex

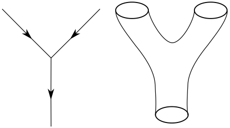


String Theory

- In String Theory, fundamental object is a string
- If strings move, their trajectory is a manifold (called **worldsheet**)



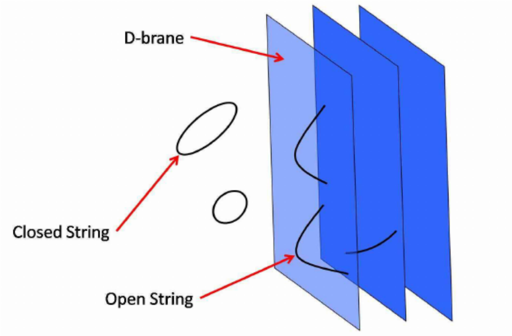
- If strings interact, their interaction differs from a vertex



- String Theory can only be consistently formulated in 10/11/26 dimensions

Branes

- In addition to strings, string theory has additional objects called branes
- Simply put, branes are places where strings can start/end



Branes and Field Theory

- Dynamics of string endpoints on a brane are like charges
- Charge dynamics on a single brane is like electromagnetism: a U(1) gauge theory
- For two branes, one can get a U(2) gauge theory
- For three branes, U(3)
- For N-branes, get U(N) $\rightarrow$ leads to SU(N) gauge theory
- A stack of N D3-branes corresponds to an SU(N) gauge theory in 3+1 dimensions

Dynamics of D-branes

- D-branes are heavy
- A stack of N D-branes will bend space-time, so we need GR to describe them
- It's simplest to assume maximal symmetry for the N D3-branes; this happens to be a field theory called $\mathcal{N} =$ SU(N) Super-Yang-Mills theory in 3+1d
- Because it's a maximally symmetric field theory, also the gravity description needs to be maximally symmetric
- For the D-branes embedded in 10 dimensions, the line element factorizes into

$$ds^2 = \mathrm{AdS}_5 \times \mathrm{S}_5 \tag{43.1}$$

AdS spacetimes

- The metric part corresponding to the S_5 factors out and does not contribute to the dynamics
- The AdS_5 part reads

$$ds^2 = L^2 \frac{-dt^2 + dx^2 + dy^2 + dz^2 + du^2}{u^2}\,, \tag{43.2}$$

which is just the 5-dimensional analogue of Anti-de-Sitter spacetime

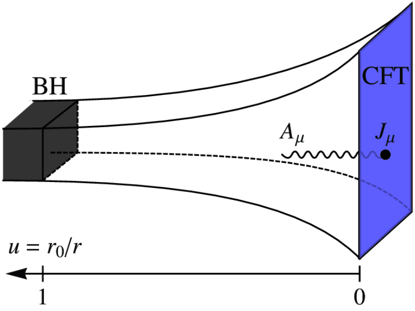
- The AdS spacetime has a curious feature at $u = 0$: it has a boundary!
- On the "conformal boundary" at $u = 0$, the metric is locally

$$g_{\mu\nu} = -dt^2 + d\vec{x}^2\,. \tag{43.3}$$

- It's Minkowski in 3+1d!

Black Branes in AdS/CFT

- So far we have talked about empty AdS – corresponding to a vacuum state on the field theory side
- However, we can consider black hole solutions in AdS
- The simplest such solution is a **black brane**: a non-compact black hole with a horizon located at $u = u_s = \mathrm{const} = \frac{1}{\pi T}$.
- Diagrammatically, the space-time looks like this:



Black Hole Thermodynamics

- Recall from lecture 33 that black holes have an entropy given by

$$S = \frac{A}{4G} \,. \tag{43.4}$$

- The only difference for black branes in AdS_5 is that the area has units of volume and $G = G_5$ is the 5-dimensional Newton constant:

$$G_5 = \frac{\pi L^3}{2N^2} \tag{43.5}$$

- The "area" of the black brane is related to u_s as

$$A = \frac{V_3 L^3}{u_s^3} = V_3 L^3 \pi^3 T^3 \tag{43.6}$$

Black Hole Thermodynamics

- Putting things together, this gives

$$S = V_3 \frac{\pi^2 T^3 N^2}{2} \tag{43.7}$$

- The entropy *density* therefore is

$$s = \frac{\pi^2 T^3 N^2}{2} \tag{43.8}$$

- This can be compared to the entropy density of $\mathcal{N} = 4$ SYM

AdS/CFT at work

- $\mathcal{N} = 4$ SYM has 8 N^2 boson and 8 N^2 fermion degrees of freedom
- For a non-interacting theory, each degree of freedom contributes the same amount to the entropy density:
- To be precise, there is $\frac{2\pi^2 T^3}{45}$ for each boson and $\frac{2\pi^2 T^3}{45} \times \frac{7}{8}$ for each fermion
- For free $\mathcal{N} = 4$ SYM one thus has

$$s_{\rm free} = \frac{2\pi^2 T^3 N^2}{45} \times 15 = \frac{2\pi^2 T^3 N^2}{3}\,. \qquad (43.9)$$

- Comparing with the AdS result (43.8, one finds

$$\frac{s}{s_{\rm free}} = \frac{3}{4}\,. \qquad (43.10)$$