Feynman Diagrams

paul.romatschke@colorado.edu

Fall 2020

Review from past lectures

- In lecture 12, the perturbative expansion of the pressure was given as

$$p = p_{\rm free} + p_{(1)} + p_{(2)} + \ldots \tag{14.1}$$

- The first-order perturbative correction was given as

$$p_{(1)} = -\frac{1}{\beta V}\langle S_I \rangle \tag{14.2}$$

- Second-order:

$$p_{(2)} = \frac{1}{\beta V}\frac{1}{2}\left(\langle S_I^2\rangle - \langle S_I\rangle^2\right) , \tag{14.3}$$

Applying Wick's theorem

- Using Wick's theorem, the first-order perturbative correction is

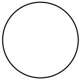
$$p_{(1)} = -\frac{3\lambda}{\beta V}\int d^4x \left(\langle\phi(x)\phi(x)\rangle\right)^2 . \tag{14.4}$$

- We will call the object $\langle\phi(x)\phi(y)\rangle$ the (free) "propagator" or (free) "two-point function"
- It represents how information gets passed from point x to point y
- We can represent it as a line:

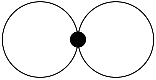
$\langle\phi(x)\phi(y)\rangle =$

Feynman Diagrams

- For the first-order pressure, we need $\langle\phi(x)\phi(x)\rangle$
- Because the arguments of ϕ are the same, the line ends at its origin
- It's a loop:

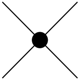
$\langle\phi(x)\phi(x)\rangle =$ 

- Because (14.4) has two propagators that close onto themselves, we have two loops;and there is a coupling λ that we'll denote as a dot; so we get

$p_{(1)} =$ 

Feynman Diagrams

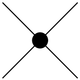
- The pictorial representation of (14.4) is called a "Feynman diagram"
- It contains all the relevant information contained in the equation
- We already know that a line means $\langle \phi(x)\phi(y) \rangle$
- Now add to this the rule that a dot means a "vertex"

 $= -\lambda$

- ...and that we have to keep track on how many ways there are to draw the diagram (the combinatorial factor)

Combinatorial Factor

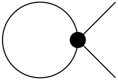
Let's do an example: Start with a single vertex:

 $= -\lambda$

Now pick one of the "legs" and connect with another "leg"

Combinatorial Factor

Let's do an example: Start with a single vertex:

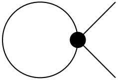


$$= -3\lambda\langle\phi(x)\phi(x)\rangle$$

Now pick one of the "legs" and connect with another "leg"
There are 3 other legs to choose from, so we get the factor three and one propagator

Combinatorial Factor

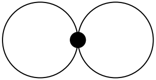
Let's do an example: Start with a single vertex:

 $= -3\lambda\langle\phi(x)\phi(x)\rangle$

Now pick another "leg" and connect it

Combinatorial Factor

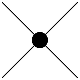
Let's do an example: Start with a single vertex:



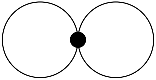
$$= -3\lambda\langle\phi(x)\phi(x)\rangle\langle\phi(x)\phi(x)\rangle$$

Now pick another "leg" and connect it
There is only one choice left for the connection, so we get a factor of 1, and another propagator

- The pictorial representation of (14.4) is called a "Feynman diagram"
- It contains all the relevant information contained in the equation
- We already know that a line means $\langle\phi(x)\phi(y)\rangle$
- Now add to this the rule that a dot means a "vertex"


$= -\lambda$

- ...and that we have to keep track on how many ways there are to draw the diagram (the combinatorial factor)
- ...a little extra thought then leads to an additional rule giving an integral, so that

$p_{(1)} =$  $= -3\lambda \frac{1}{\beta V} \int_x \left(\langle\phi^2(x)\rangle\right)^2$

Feynman Diagrams

- Feynman diagrams are a shorthand notation for integrals in perturbation theory
- To decode Feynman diagrams, one must first calculate the so-called "Feynman rules"
- Feynman rules can be formulated in coordinate-space (x-space) or momentum space
- Expressing perturbation theory in Feynman diagrams can lead to simplifications (mostly at higher order in perturbation theory)
- It is hard (but not impossible) to express *non-perturbative* results using Feynman diagrams
- As a consequence, Feynman diagrams are most useful in perturbation theory (whenever the coupling is very weak, $\lambda \ll 1$)

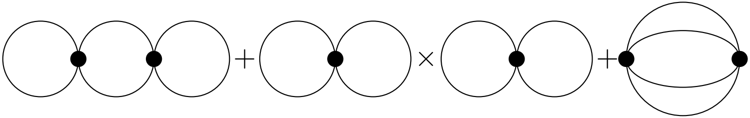
Example of simplification using Feynman Diagrams

- Let us point out one example where Feynman diagrams simplify a perturbative calculation
- Recall from (14.3) that the pressure to second order in perturbation theory is

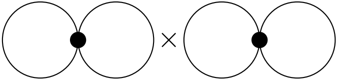
$$p_{(2)} = \frac{1}{\beta V}\frac{1}{2}\left(\langle S_I^2\rangle - \langle S_I\rangle^2\right)$$

- Expanding $\langle S_I^2\rangle$ in Feynman diagrams we have

$\langle S_I^2\rangle =$



Disconnected diagrams

We call  a “disconnected” diagram

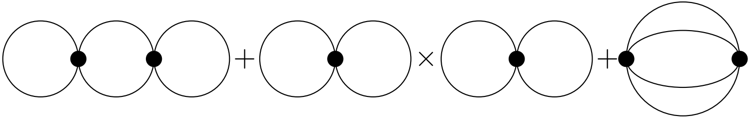
This is because there is no propagator connecting the two vertices, so the corresponding integral factorizes

The term “disconnected diagram” generalizes to all diagrams where at least one vertex is not connected to the rest of the diagram

Example of simplification using Feynman Diagrams

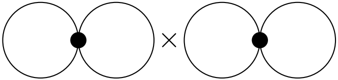
- Expanding $\langle S_I^2 \rangle$ in Feynman diagrams we have

$\langle S_I^2 \rangle =$



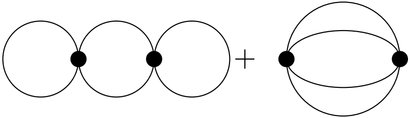
- From first-order perturbation theory, we have

$\langle S_I \rangle^2 =$



- Taken together, for $p_{(2)}$ the “disconnected” diagrams exactly cancel!

$p_{(2)} =$



Feynman Diagrams

- The cancellation of the disconnected diagrams in $p_{(2)}$ is not a coincidence
- One can show that for physical observables (such as the pressure), *all* disconnected diagrams cancel
- As a consequence, one only needs to consider *connected* diagrams when calculating observables in perturbation theory
- This is a major simplification when performing high order perturbative calculations!