The O(N) Vector Model III

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Review

- In lecture 26, we solved the O(N) vector model in the large N limit
- In lecture 27, we renormalized the O(N) model for $m = 0$, finding

$$p(T,\lambda) = \frac{Nx^2}{16\pi^2}\left[\ln\left(\frac{\bar{\mu}^2 e^{\frac{3}{2}}}{2x}\right) + \frac{2\pi^2}{\lambda_{\rm phys}(\bar{\mu})}\right] - NJ_B(T,\sqrt{2x}) \quad (28.1)$$

  for the pressure of the O(N) model
- Here x is the location of the saddle defined by the condition

$$\frac{x}{8\pi^2}\left[\frac{2\pi^2}{\lambda_{\rm phys}(\bar{\mu})} + \ln\left(\frac{\bar{\mu}^2 e^1}{2x}\right)\right] - I_B(T,\sqrt{2x}) = 0 \quad (28.2)$$

- We will evaluate this result at finite temperature in this lecture

Finite Temperature

- Recall that the exact running coupling for the O(N) model for $N \gg 1$ is given by

$$\lambda_{\rm phys}(\bar\mu) = \frac{2\pi^2}{\ln\frac{\mu_0^2}{\bar\mu^2}}\,, \tag{28.3}$$

  where μ_0 is the Landau pole of the theory
- Close to the Landau pole, the QFT breaks down, so we can only probe scales $\bar\mu \ll \mu_0$
- At finite temperature, we can choose to express T in units of μ_0

$$T = \mu_0 e^{-\chi}\,, \tag{28.4}$$

  with $\chi \gg 1$
- Of course, we can also put $\mu_0 = e^{\chi} T$

Finite Temperature Pressure

- Using the explicit coupling (28.3) and (28.4) the pressure and saddle point condition in the O(N) model are given by

$$p(T) = N\left[\frac{x^2}{16\pi^2}\ln\left(\frac{T^2 e^{\frac{3}{2}+2\chi}}{2x}\right) - J_B(T,\sqrt{2x})\right]\,, \tag{28.5}$$

$$\frac{x}{8\pi^2}\ln\left(\frac{T^2 e^{1+2\chi}}{2x}\right) - I_B(T,\sqrt{2x}) = 0\,, \tag{28.6}$$

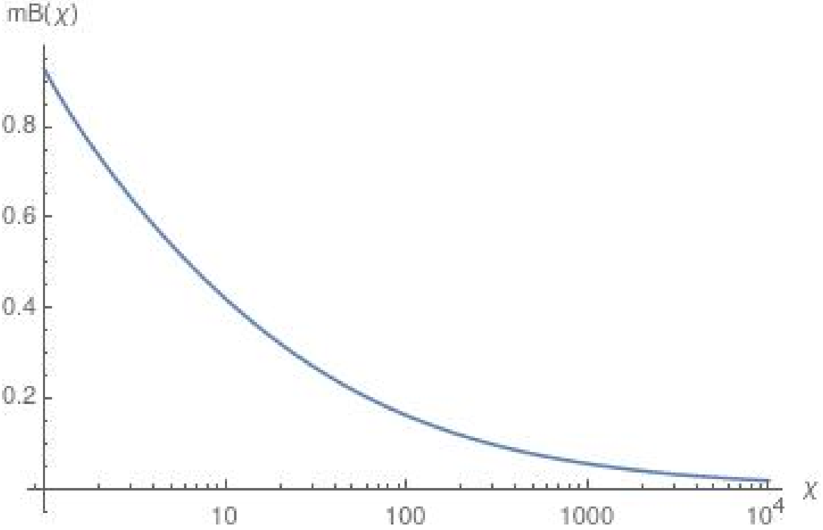
where $J_B(T,m) = -\frac{m^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_2(nm\beta)}{n^2}$ and $I_B(T,m) = \frac{mT}{2\pi^2}\sum_{n=1}^{\infty}\frac{K_1(nm\beta)}{n}$, see homework problem 11

- Note that p(T) is *independent* from $\bar{\mu}$, as it should be!

Saddle Point Condition

- Since there are no other mass-scales in the problem, and since $[x] = 2$, the saddle point location x has to scale as temperature squared
- We set $x = m_B^2 T^2$ such that the saddle point condition becomes a condition for $m_B(\chi)$
- Large values of χ corresponds to small coupling (low temperature), while small χ corresponds to big coupling (high temperature), cf. (28.3)
- Solving the saddle point condition (28.6) numerically leads to $m_B(\chi)$
- We can convert results back to $m_B(T)$ by using $T = \mu_0 e^{-\chi}$

The scalar in-medium mass



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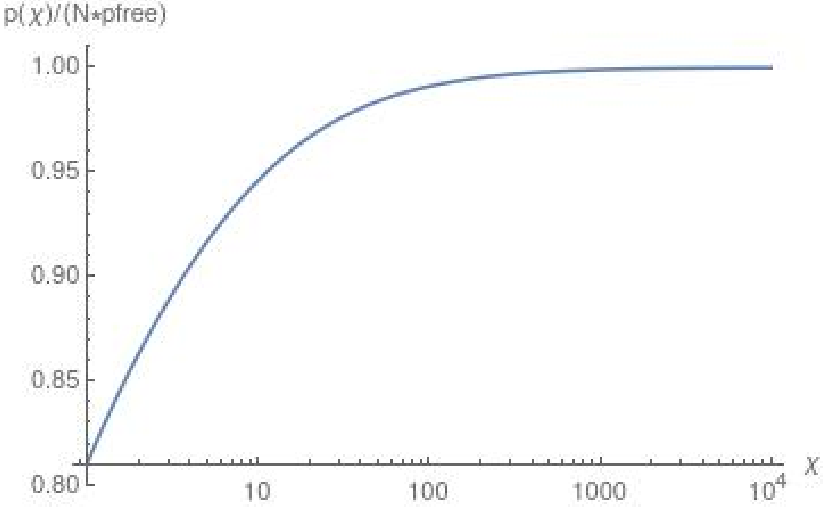

The O(N) Model

- We get non-perturbative results for the pressure in the O(N) model
- These results are well-behaved and physically sensible for $\chi \gg 1$
- In terms of the original coupling constant (28.3), this includes the regime where

$$\lambda_{\rm phys}(\bar{\mu}) = \frac{2\pi^2}{\ln \frac{\mu_0^2}{\bar{\mu}^2}} = \frac{2\pi^2}{\ln \frac{T^2 e^{2\chi}}{\bar{\mu}^2}} \gg 1\,, \tag{28.7}$$

  choosing e.g. $\bar{\mu} = 2\pi T$
- We get access to the **strong coupling** regime, but the Landau pole prevents us from accessing $\lambda_{\rm phys} \to \infty$

The O(N) Model in 2+1d

- If we *reduce* the number of space dimensions to two, we can study the O(N) model in 2+1 dimensions
- Choosing the coupling to be of the form $(\vec{\phi} \cdot \vec{\phi})^3$, the 3d O(N) model becomes a **pure conformal field theory**, since the conformal anomaly vanishes for all couplings
- The 3d O(N) model is very special, and does not have a Landau pole
- We can access $\lambda_{\rm phys} \to \infty$ directly from the field theory!
- For advanced students, I suggest taking a look at https://arxiv.org/pdf/1904.09995.pdf