Photon Propagator

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Review

- In lecture 43, we derived the amputated photon-electron vertex in QED
- A part of the result was the photon propagator

$$G_{\mu\nu}(x) \equiv \langle A_\mu(x) A_\nu(0) \rangle \tag{45.1}$$

  in the free theory
- We discussed the U(1) partition function in lecture 36, which we will use as a starting point

The Partition Function

- The partition function for the U(1) gauge field was given in lecture 36 as

$$Z = \int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c e^{-S_{\rm gauge}-S_{\rm gf}-S_{\rm ghost}}\,. \tag{45.2}$$

- Here

$$S_{\rm gauge} = \frac{1}{4}\int_x F_{\mu\nu}^2\,,\quad S_{\rm gf} = \frac{1}{2\xi}\int_x G^2[A]\,,\quad S_{\rm ghost} = \int_x \bar{c}\frac{\partial G[A]}{\partial\alpha}c\,, \tag{45.3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $G[A]$ an arbitrary gauge-fixing condition, and ξ an arbitrary gauge-fixing parameter

The Propagator

- Similar to the partition function, the free photon propagator is given by

$$G_{\mu\nu}(x) = \frac{\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c A_\mu(x) A_\nu(0) e^{-S_{\rm gauge}-S_{\rm gf}-S_{\rm ghost}}}{Z} \tag{45.4}$$

- Not surprisingly, the form of the propagator depends on the choice of the gauge-fixing condition $G[A]$
- In the following, we will consider so-called covariant gauges, e.g.

$$G[A] = \partial_\mu A_\mu \,. \tag{45.5}$$

Covariant Gauges

- Within the class of covariant gauges, the ghosts decouple for a U(1) gauge theory as in lecture 36
- We have for the propagator

$$G_{\mu\nu}(x) = \frac{\int \mathcal{D}A A_\mu(x) A_\nu(0) e^{-S_{\rm gauge} - S_{\rm gf}}}{Z_A}\,, \tag{45.6}$$

where $Z_A = \int \mathcal{D}A e^{-S_{\rm gauge} - S_{\rm gf}}$

Fourier Transform

- We can perform a Fourier transform of the gauge field:

$$A_\mu(x) = \frac{T}{V}\sum_K e^{iK\cdot x}\tilde{A}_\mu(K) \tag{45.7}$$

- The gauge-field part of the partition function then becomes (cf 36.9)

$$Z_A = \int \mathcal{D}\tilde{A} e^{-\frac{T}{2V}\sum_K \tilde{A}_\mu(K) M_{\mu\nu}(K)\tilde{A}_\nu(-K)}\,, \tag{45.8}$$

where

$$M_{\mu\nu}(K) = K^2\delta_{\mu\nu} - K_\mu K_\nu + \frac{1}{\xi}K_\mu K_\nu \tag{45.9}$$

Propagator

- The Fourier-transformed propagator becomes

$$G_{\mu\nu}(P) = \frac{T}{V}\sum_Q \frac{\int \mathcal{D}\tilde{A}\tilde{A}_\mu(P)A_\nu(Q)e^{-\frac{T}{2V}\sum_K \tilde{A}_\mu(K)M_{\mu\nu}(K)\tilde{A}_\nu(-K)}}{Z_A}, \tag{45.10}$$

- Since the integral is Gaussian, we have

$$\frac{\int \mathcal{D}\tilde{A}\tilde{A}_\mu(P)A_\nu(Q)e^{-\frac{T}{2V}\sum_K \tilde{A}_\mu(K)M_{\mu\nu}(K)\tilde{A}_\nu(-K)}}{Z_A} = \frac{V}{T}\delta(P+Q)M^{-1}_{\mu\nu}(P) \tag{45.11}$$

- Hence

$$G_{\mu\nu}(P) = M^{-1}_{\mu\nu}(P)\,. \tag{45.12}$$

Propagator

- We can calculate the inverse of the matrix $M_{\mu\nu}$ using the projectors, cf (36.11)

$$P^T_{\mu\nu} = \delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}\,, \quad P^L_{\mu\nu} = \frac{K_\mu K_\nu}{K^2}\,. \tag{45.13}$$

- From $M = K^2 P^T + \frac{K^2}{\xi} P^L$ we have

$$M^{-1}_{\mu\nu}(K) = \frac{1}{K^2} P^T_{\mu\nu} + \frac{\xi}{K^2} P^L_{\mu\nu} \tag{45.14}$$

- Therefore, we have

$$G_{\mu\nu}(P) = \frac{\delta_{\mu\nu}}{P^2} + (\xi - 1)\,\frac{P_\mu P_\nu}{P^4} \tag{45.15}$$

Feynman Gauge

- Recall that the parameter ξ is arbitrary
- Physical observables cannot depend on ξ, so independence wrt ξ can be used as a check on calculations
- Conversely, if we trust our ability to correctly do calculations, we may choose a convenient value of ξ
- A particularly convenient value is $\xi = 1$, also called "Feynman gauge"
- In Feynman gauge, the propagator becomes

$$G^{\xi=1}_{\mu\nu}(P) = \frac{\delta_{\mu\nu}}{P^2} . \tag{45.16}$$