g-2, part IV

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Review

- In lecture 42, we discussed that g-2 is related to the connected and amputated photon-electron vertex $\Gamma_\mu^{\rm conn.,amp.}$
- In lecture 46, we derived the result

$$\Gamma_\mu^{(3),\rm conn.,amp.}(P,P') = -2ie^3\int_0^1 dx_1 dx_2 \int_K \frac{N_\mu}{D^3}\,, \tag{47.1}$$

- The explicit expressions for N_μ, D can be found in eqns. (46.18) in lecture 46
- In this lecture, we will continue with the evaluation of the vertex

K-integrations

- Let us first study the contribution $\not{K}\gamma_\mu\not{K}$ in N_μ
- We have

$$\int_K \frac{\not{K}\gamma_\mu\not{K}}{D^3} = \int_K \frac{-K^2\gamma_\mu + 2\not{K}K_\mu}{D^3} \tag{47.2}$$

- Since D depends on K only as K^2, and since the integration measure is Lorentz covariant, we must have

$$\int_K \frac{K_\mu K_\nu}{D^3} = A\delta_{\mu\nu}\int_K \frac{K^2}{D^3}\,, \tag{47.3}$$

  with A a constant that has to be determined.
- Contracting both sides with $\delta_{\mu\nu}$, we have in four dimensions

$$\int_K \frac{K^2}{D^3} = 4A\int_K \frac{K^2}{D^3}\,, \tag{47.4}$$

  so $A = \frac{1}{4}$

K-integrations

- As a consequence, we have
- We have

$$\int_K \frac{\not{K}\gamma_\mu \not{K}}{D^3} = \left(-1+\frac{2}{4}\right)\gamma_\mu \int_K \frac{K^2}{D^3} = -\frac{\gamma_\mu}{2}\int_K \frac{K^2}{D^3}\,. \tag{47.5}$$

- The denominator is of the form

$$D = K^2 + A\,, \tag{47.6}$$

where A is independent from K

- Adding and subtracting A in the numerator gives

$$\int_K \frac{\not{K}\gamma_\mu \not{K}}{D^3} = -\frac{\gamma_\mu}{2}\int \frac{1}{[K^2+A]^2} + \frac{\gamma_\mu}{2}\int_K \frac{A}{[K^2+A]^3} \tag{47.7}$$

K-integrations

- The first term in this expression is divergent, and needs to be regulated
- Using dimensional regularization in $D = 4 - 2\varepsilon$, we have

$$\int \frac{1}{[K^2 + A]^2} \simeq \frac{1}{\varepsilon} - \ln \frac{A}{\mu^2} \tag{47.8}$$

- For completeness, we also have

$$\int \frac{A}{[K^2 + A]^3} \simeq \frac{1}{2}\,. \tag{47.9}$$

- Therefore,

$$\int_K \frac{\not{K}\gamma_\mu \not{K}}{D^3} \simeq -\gamma_\mu \left(\frac{1}{\varepsilon} - \ln \frac{A}{\mu^2} - \frac{1}{2}\right)\,. \tag{47.10}$$

K-integrations: remarks

- To properly calculate the finite-part of this contribution, we have to generalize the Clifford algebra relations from four dimensions to $D = 4 - 2\varepsilon$
- However, it turns out we won't need it for g-2
- The reason is that we can choose a renormalization scheme such as on-shell, where zero-momentum corrections to the electron-photon vertex are vanishing
- Specifically, for photon momentum $Q_\mu = P'_\mu - P_\mu$, we expect that after renormlization

$$\Gamma_\mu^{(3),,\mathrm{conn.},\mathrm{amp.},\mathrm{ren}}(P' = P, P) = 0\,, \qquad (47.11)$$

with $P^2 = -m^2$, $P'^2 = -m^2$ (on-shell fermions).

K-integrations: remarks

- From lecture 46, we have for the explicit form of A:

$$A = P^2 x_1(1-x_1) + P'^2 x_2(1-x_2) + m^2(x_1+x_2) - 2x_1x_2 P\cdot P' \,. \quad (47.12)$$

- Noting that

$$2P\cdot P' = -(P-P')^2 + P^2 + P'^2 = -Q^2 - 2m^2 \quad (47.13)$$

we find

$$\begin{aligned} A &= -m^2 x_1(1-x_1) - m^2 x_2(1-x_2) + m^2(x_1+x_2) \\ &\quad -x_1x_2(-Q^2-2m^2)\,, \\ &= m^2(x_1+x_2)^2 + x_1x_2 Q^2 \,. \end{aligned} \quad (47.14)$$

K-integrations: remarks

- Returning to the result from K-integration (47.10), after on-shell renormalization we have

$$\int_K \frac{\not{K}\gamma_\mu \not{K}}{D^3}\Big|_{\rm ren} \simeq \gamma_\mu \ln \frac{m^2(x_1+x_2)^2 + x_1 x_2 Q^2}{m^2(x_1+x_2)^2}\,,$$
$$\simeq \gamma_\mu \frac{x_1 x_2 Q^2}{m^2(x_1+x_2)^2} + \mathcal{O}(Q^4) \qquad (47.15)$$

- For $g-2$ we are looking for the structure factor F_2, which involves a term *linear* in photon momentum
- As a consequence, there is no contribution to $g-2$ from $\int_K \frac{\not{K}\gamma_\mu \not{K}}{D^3}$