Name: DUE ON 16.03.2015

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# Homework - 1

1. Try to get a feel for the units that are used to measure distances in astrophysics by filling in the gaps. Use the scientific notation for large and small numbers and give at most 2 significant digits, i.e. $*.* \times 10^{\pm n}$.

   (a) The distance between the earth and the moon is .................... light-seconds.

   (b) The distance between the earth and the sun is one astronomical unit 1 $au$ = .................... light minutes = ....................$pc$.

   (c) 1 light-year = 1 $ly$ = .................... meters = ....................$pc$

   (d) The distance between the sun and the nearest star is .................... $pc$.

   (e) The distance between the earth and the center of the milky way is .................... $pc$ = .................... $kpc$.

   (f) The diameter of the milky way is ....................$kpc$.

   (g) The distance between the earth and the Andromeda galaxy which is the closest galaxy is ....................$kpc$.

   The universe is homogeneous and isotropic on scales larger than a few Mpc.

2. A simple toy universe: Let us solve the Friedmann equations in the presence of only a cosmological constant:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a^2} = \frac{\Lambda}{3}\,, \tag{1}$$

$$\left[\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3}\right]. \tag{2}$$

   It is sufficient to solve the first equation since the second one is implied by the first.

   (a) Set $K = 0$ and assume that $\Lambda > 0$. Determine $a(t)$ in an expanding universe. On dimensional grounds the cosmological constant should be $\Lambda = \lambda \cdot \frac{c^5}{\hbar G} \approx \lambda \cdot 10^{86}/s^2$. We don't know of any reason why $\lambda$ should be particularly small so let us set $\lambda = 1$. Assume two atoms are initially separated by a distance of $d_0 = 10^{-10}m$. How long does it take in this expanding universe until their distance equals the distance between the earth and the center of our galaxy? You just discovered the cosmological constant problem. In our universe $\lambda$ turns out to be incredible small $\lambda \approx 10^{-120}$.

   (b) Solve the Friedmann equations for $K = -1$ and a negative cosmological constant. Fix the initial condition so that $a(t = 0) = 0$. Sketch $a(t)$, remembering that the scale factor is determining distances and therefore has to satisfy $a(t) \geq 0$. The fate of such a universe is often called a big crunch.