Name: DUE ON 27.04.2015

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Homework - 5

1. The largest particle accelerator in the world, the Large Hadron Collider at CERN can reach energies of roughly $1TeV$. So we understand and can experimentally test physics up to this energy scale. This means in turn that we essentially understand the evolution of our universe up to this energy scale very well.
   The matter-radiation equality happened at redshift $z = 3400$, which corresponds to $t_{eq} = 6 \times 10^4$ years after the big bang. Before that our universe was radiation dominated $a(t) \approx a_{eq}\sqrt{t/t_{eq}}$ for $t < t_{eq}$. Determine the time *in seconds* at which $T_{CMB} = 1TeV$.

2. The early universe was transitioning from a radiation to a matter dominated phase. Let us derive an analytic solution for the scale factor for such a two component universe.

   (a) Rewrite the Friedmann equations (given in equations (1) and (2) in the lecture 2 notes) using conformal time.

   (b) Now assume only radiation and matter as source terms (i.e. set $K = 0$). Check that the second equation involving $a''(\tau)$ is not sourced by the radiation but only couples to matter. Simplify this equation using that $\rho_m a(\tau)^3 = const. = \frac{1}{2}\rho_{eq} a_{eq}^3$, where the subscript eq refers to the time of matter and radiation equality.

   (c) Solve this trivial differential equation. Fix one of the two integration constants by demanding that $a(\tau = 0) = 0$.

   (d) Fix the second integration constant by using the first Friedmann equation and the fact that

   $$\rho = \rho_m + \rho_{rad} = \frac{\rho_{eq}}{2}\left[\left(\frac{a_{eq}}{a}\right)^3 + \left(\frac{a_{eq}}{a}\right)^4\right]. \tag{1}$$

   Express your answer in terms of a_{eq} and τ/τ_* with

   $$\tau_* \equiv \sqrt{\frac{3}{\pi G \rho_{eq} a_{eq}^2}} = \frac{\tau_{eq}}{\sqrt{2} - 1}. \tag{2}$$