

Homework - 8

1. (a) Show explicitly that the Boltzmann equation as given in equation (5) in the lecture 8 notes can be rewritten as given in equation (6).
 - (b) Assume that $n_i = n_i^{eq}$ for $i = 2, 3, 4$. According to equation (5) how does $n_1 a^3$ evolve in the three cases $n_1 < n_1^{eq}$, $n_1 > n_1^{eq}$ and $n_1 = n_1^{eq}$?
2. Consider the following process in our early universe at $T \gg m_f$:

$$f + \bar{f} \leftrightarrow \gamma + \gamma, \quad (1)$$

where f denotes a fermion and \bar{f} the corresponding anti-fermion.

- (a) If the fermion chemical potential is given by μ_f , what is the anti-fermion chemical potential $\mu_{\bar{f}}$ in thermal and chemical equilibrium?
- (b) Assume that the fermions are quarks in the early universe. After baryogenesis the number of quarks minus anti-quarks is non-zero and does not change. Calculate the difference between the quark and anti-quark number densities for non-zero chemical potential

$$n_q - n_{\bar{q}} = \frac{g}{2\pi^2} \int_0^\infty dp p^2 \left(\frac{1}{e^{\frac{p-\mu}{T}} + 1} - \frac{1}{e^{\frac{p+\mu}{T}} + 1} \right). \quad (2)$$

You see that the exact expression is fairly simple and that it is indeed non-vanishing for non-zero chemical potential μ .

Since $a^3(n_q - n_{\bar{q}}) = T^{-3}(n_q - n_{\bar{q}})$ is constant, we find that μ is a function of the temperature, which is why one usually works with ratios of number densities in which the chemical potentials cancel. (The dependence of μ on T in the non-relativistic case, is more complicated.)

Hint: Don't worry about the value of g . You can use that

$$\int_0^\infty dx \frac{x^2}{e^{x-c} + 1} = -2Li_3(-e^c)$$

and that the polylogarithm satisfies

$$Li_3(-e^{-x}) - Li_3(-e^x) = \frac{x}{6}(\pi^2 + x^2).$$

3. The early universe is radiation dominated.
 - (a) Determine the scaling of the time t as a function of the temperature T .
 - (b) At a temperature of $T = 1MeV$ our universe is approximately 1.4 seconds old. Determine the time in seconds at which the deuterium abundance is comparable to the abundance of protons, i.e. at which $T \approx .066MeV$.