Name: DUE ON 15.06.2015

Matrikelnummer: **NO CLASS ON 08.06.2015**

Homework - 9

1. The neutral Hydrogen and Helium that fills the universe after recombination starts to clump due to small inhomogeneities. This leads to the formation of the first stars a few hundred million years after the big bang. The nuclear fusion in stars releases energy in terms of photons. This energy is approximately $7MeV$ per Hydrogen atom.

   (a) From the binding energy of Hydrogen which is $13.6eV$ get a simple estimate of the amount of Hydrogen that needs to undergo nuclear fusion in order to release enough energy to ionize all the Hydrogen in the universe. [1]

   (b) The first stars quickly lead to a complete reionization of the universe. From section 3.3 in the lecture 8 notes, derive the maximal temperature T_{max} at which this could have happened without any substantial impact on the cosmic microwave background. (The temperature during reionization is roughly $T \approx 4meV < T_{max}$.)

2. The horizon problem of the cosmic microwave background:

   In Homework 5 you derived the solution for the scale factor in the presence of matter and radiation

$$a(\tau) = a_{eq}\left[\left(\frac{\tau}{\tau_\star}\right)^2 + 2\frac{\tau}{\tau_\star}\right], \qquad \tau_\star = \frac{\tau_{eq}}{\sqrt{2}-1}. \tag{1}$$

   We neglect the cosmological constant in this problem and use the above expression to describe our universe up until today. Recall that the redshift at matter-radiation equality is $z_{eq} = 3400$ and the redshift at photon decoupling is $z_{CMB} = 1100$. We will work in conformal time and express everything in terms of $\tau_\star$.

   (a) Calculate τ_D as a function of $\tau_\star$.

   (b) Calculate the particle horizon $d_H(\tau_D)$ at photon decoupling $\tau = \tau_D$ and express it as a function of a_0 and $\tau_\star$ only.

   (c) The CMB photons we observe today were emitted from a huge sphere that has the earth at its center. Calculate the physical radius of this sphere.

   (d) Get an estimate for the number of causally disconnected patches in the CMB by calculating the surface area of the sphere in (c) and dividing it by the cross-sectional area of a causally connected patch $\pi(d_H(\tau_D)\, a_0/a(\tau_D))^2$ with $d_H(\tau_D)$ from (b). The factor $(a_0/a(\tau_D))^2$ is taking into account the growth of the causally connected patch until today.

[1] Note, that the actual fraction of Hydrogen that needs to undergo nuclear fusion to ionize the universe is larger by a factor of ten than this rough estimate. The reason is that the electron and proton can recombine several times so that the Hydrogen needs to be ionized several times and also some of the energy that is released during the nuclear fusion is carried by photons whose energy is below the $13.6eV$ ionization energy of Hydrogen.