Name: DUE NEXT CLASS

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# Homework - 10

1. The scale factor during inflation behaves likes $a(t) \propto e^{H_{inf} t}$, where $H_{inf}$ is the approximately constant Hubble scale during inflation. We need roughly 60 e-folds of expansion in order to solve the horizon and flatness problem. How long would inflation have to at least last, if $H_{inf} = 10^{14} GeV$ and how long if $H_{inf} = 100 TeV$?

2. Power-law potentials: Assume that the inflaton potential is given by $V(\phi) = \lambda M_P^4 \left(\frac{\phi}{M_P}\right)^p$, where $p > 0$ is not necessary an integer.

   (a) Calculate the slow-roll parameters $\epsilon_V$ and $\eta_V$ for this potential.

   (b) When does inflation end, i.e. when is the smaller of the two slow-roll parameters equal to one? (Do a case analysis $p \leq 2$ and $p \geq 2$.)

   (c) Using equation (26) in the lecture 10 notes, calculate the initial displacement $\phi_i$ that is needed in order to get 60 e-folds of inflation.

   For the two problems below restrict to the case $p \leq 2$.

   (d) The current experimental bounds are $r \equiv 16\epsilon_V < .11$ and $n_s \equiv 1 - 6\epsilon_V + 2\eta_V = .968 \pm .010$ (95% CL). Assume 60 e-folds of inflation and evaluate $\epsilon_V$ and $\eta_V$ at $\phi_i$. From the upper bound on $r$ calculate the corresponding upper bound on $p$. From the bound on $n_s - 1$ calculate a lower bound on $p$.

   (e) The energy scale during inflation is given by $V_{inf} \approx 2 \times 10^{16} GeV \left(\frac{r}{.1}\right)^{\frac{1}{4}}$. Based on the bound on $p$ calculated above, find the energy range for these models.

   *N*ote: The highest energy scales we can reach in experiments are around $10^3 - 10^4\, GeV$, so that an experimental discovery of any of these power-law models would open up a completely new frontier in high energy physics.

3. Sensitivity to Planck suppressed operators: Without a UV complete theory of gravity, we cannot calculate higher order corrections to the scalar potential. Usually this is no problem since these corrections are suppressed by a high energy scale (the cut-off scale of our low energy effective action). Let us assume that this scale is given by the highest plausible value, the Planck scale $M_P$. Assume there is such a correction that modifies the scalar potential so that it becomes

$$V_{cor} = V(\phi)\left(1 + c\frac{\phi^2}{M_P^2}\right), \qquad (1)$$

   where $V(\phi)$ is the original scalar potential and $c$ some unknown constant. Clearly such corrections are crucial in large field models with $\Delta\phi \gtrsim M_P$. Calculate the leading order correction to $\eta_V$ in a small field model with $\phi \ll M_P$. You find that $\eta_{V_{cor}}$ depends on $c$ so that inflation is sensitive even to Planck suppressed corrections which makes model building extremely challenging.