Cosmology and particle physics

Lecture notes

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Lecture 5 The thermal universe - part I

In the last lecture we have shown that our very early universe was in a very hot and dense state. During the expansion of the universe this hot 'soup' cooled and underwent a variety of interesting transitions. In the next few lectures we will discuss the thermodynamical evolution of our Universe from a split second after the big bang until the release of the cosmic microwave background 380,000 years after the big bang.

We will from now on set the Boltzmann constant equal to one $k_B = 1$ and measure temperatures in eV.

1 The universe in thermal equilibrium

Currently the world's largest particle accelerator, the Large Hadron Collider, does experiments at an energy of several hundred GeV up to a few TeV. So we understand particle physics up to this energy scale very well. The particles with masses below this scale and their interactions are described by the so called standard model of particle physics, which is a particular quantum field theory. In addition to so called quarks that are the constituents of the more familiar protons and neutrons, there are leptons like the electron and the electron neutrino. These quarks and leptons come in three copies that are called families. In addition to all these fermions there are also bosons: The Higgs particle is a scalar (i.e. spinless) and the forces are mediated via vector bosons: the photon (electro-magnetic force), the gluons (strong force) and the Z and W bosons (weak force). All these particles and their masses are shown in figure 1.

Note, that the standard model or particle physics neglects gravity entirely. This is very well justified in most regimes of interest to elementary particle physics but it tells us that in order to describe our entire universe, we need another theory that unifies quantum field theories with gravity in a so called theory of everything.

General relativity that we are using in this course to describe the evolution of our universe is likewise incomplete since it is a classical theory and it inevitably breaks down near the Planck scale M_P which is given by

$$M_P = \frac{1}{\sqrt{8\pi G}} = 2.435 \times 10^{18} GeV \,. \tag{1}$$

Since the universe was at higher and higher temperatures/energies the further back in time we are going, we reach a point at which general relativity cannot be used anymore. This in particular means we cannot use general relativity to understand the actual beginning of our universe, i.e. the big bang. It is also unclear whether we will ever be able to get experimental

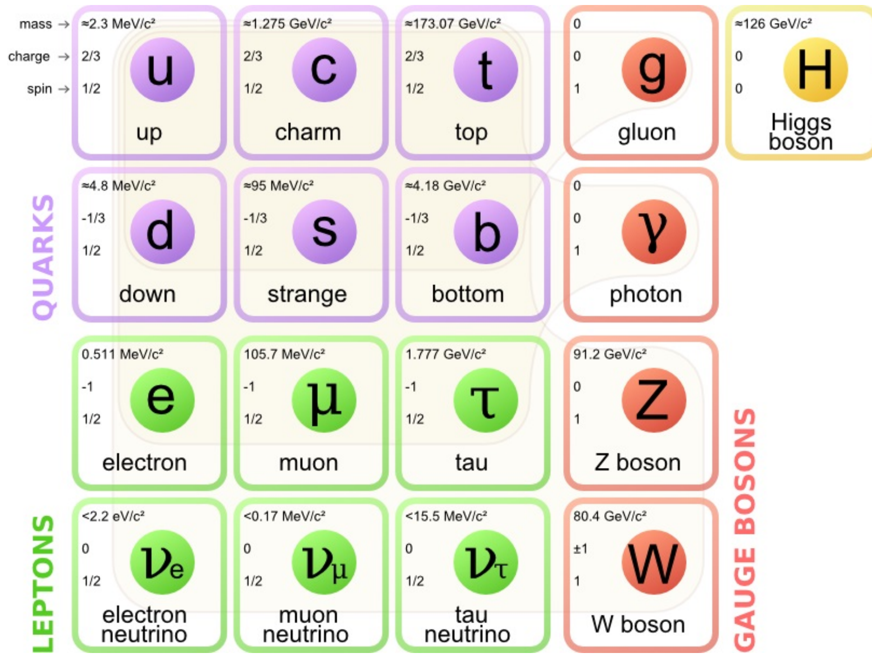


Figure 1: The known particles in our universe (taken from Wikipedia).

insight into the physics that caused the big bang. However, between the energies of less than an eV at the time the CMB was released until the energies studied in particle accelerators of a few TeV we have 12 order of magnitudes of well understood physics to discuss and from the TeV range up to the Planck scale we will discuss another 15 orders of magnitude in energy of slightly more speculative physics. There are also ideas for theories of quantum gravity that go beyond general relativity and might allow us to theoretically understand the initial singularity that arises in general relativity at the beginning of our universe.

In the early universe at temperatures above a few hundred GeV all standard model particles will have energies that are much larger than their rest mass:

$$E(p) = \sqrt{m^2 + p^2} \approx p\,. \tag{2}$$

This means that they do not behave like non-relativistic (pressureless) matter but rather like radiation (i.e. like for example photons for which $E = p$). Since the masses are negligible in this era, there is only one scale in the standard model which is the rate of interactions Γ, i.e. the number of interaction per time. In principle this rate can be different for the different particles but we neglect this for the rough estimates in this section. In our expanding universe there is one more length or time scale set by the Hubble scale H. If the particles interact a lot without feeling the expansion of the universe, then they will be in local equilibrium. This would mean that

$$\Gamma \gg H\,. \tag{3}$$

If the above equation is true, then we can use equilibrium thermodynamics to describe our universe. We would therefore like to estimate during which temperatures/energies the above is expected to be true.

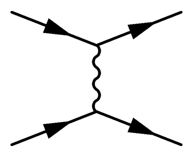
The particle interaction rate can be written as

$$\Gamma \equiv n\sigma v\,, \tag{4}$$

where n is the number density, i.e. the number of particles per volume, σ is the interaction cross-section and v is the average velocity of the particles. Since, as we argued above, all particles are highly relativistic for $T \gg 100GeV$, we have $v \approx c = 1$. The only dimensionful quantity is the temperature T, that has the dimension of an energy which is the same as an inverse length. So we find for the number density and the cross section

$$n \sim T^3\,, \qquad \sigma \sim T^{-2}\,. \tag{5}$$

For the cross section we can be more precise. Two particles interact dominantly via the exchange of one of the gauge bosons (that are all massless above $100GeV$). We often write this in terms of Feynman diagrams that use straight lines to indicate fermions and wiggly lines to describe a gauge boson:



The interaction cross section is the square of such a diagram so that it goes like the fourth power of the interaction strength between the fermion and the gauge boson. This interaction strength is usually called $\sqrt{\alpha}$, which gives

$$\sigma \sim \frac{\alpha^2}{T^2}\,. \tag{6}$$

Putting this together we find the following scaling of the interaction rate

$$\Gamma \approx n\sigma \approx \alpha^2 T\,. \tag{7}$$

The actual value of α depends on the particular force with which the particles interact as well as the energy scale. However, at high energies the interaction strength of all forces seems to become almost the same, which hints at a unification of all force in a so called grand unified theory (GUT). At this GUT scale the energy is approximately $10^{16}GeV$ and the value of α is $\alpha \approx .05$.

As we have seen last time, our early universe was dominated by radiation. [1] This means that

$$H^2 = \frac{\rho}{3M_P^2} \sim \frac{1}{a(t)^4 M_P^2} \sim \frac{T^4}{M_P^2} \qquad \Rightarrow \qquad H \sim \frac{T^2}{M_P}\,. \tag{8}$$

Putting this together we find that

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_P}{T} \sim \frac{10^{16}GeV}{T}\,, \tag{9}$$

which means for roughly $100GeV \ll T \ll 10^{16}GeV$ we have $\Gamma \gg H$. So our early and hot universe was in a state of local equilibrium and we can describe it using equilibrium thermodynamics.

[1] This is also plausible from the above discussion that showed that all the standard model particles behaved like radiation instead of matter in the early universe.

Baryogenesis

In the following we will describe the cooling of our universe starting from a 'soup' of matter and photons at a temperature of a few hundred GeV, using our knowledge of particle physics and thermodynamics. However, before we do that let us mention a puzzle: In a very hot universe we can create particle-anti-particle pairs from photons, denoted γ. For example for the electron e^- and the positron e^+ we can have the reversible process

$$e^- + e^+ \leftrightarrow \gamma + \gamma \,. \tag{10}$$

In an expanding universe we know that the photons 'loose' energy due to the redshift. This means that there is a certain point at which the two photons on the right won't have enough energy to create an electron-positron pair. At that moment the above process should only go in one direction

$$e^- + e^+ \rightarrow \gamma + \gamma \,. \tag{11}$$

If the early universe has an equal number of particles and anti-particles then eventually we would expect that all particles and anti-particles annihilate and leave a universe filled with photons. However, in our universe there is an asymmetry between matter and anti-matter, so that our universe ended up with matter and not just radiation. This asymmetry can be quantified by the ratio between the number of baryons (protons and neutrons) and photons in our current universe. Observation tell us that today

$$\frac{n_b}{n_\gamma} \sim 10^{-9} \,, \tag{12}$$

while the same ratio for anti-baryons seems to be essentially zero. There is no mechanism inside the standard model of particle physics that can explain this so called baryogenesis, i.e. the observed matter-anti-matter asymmetry, so we will simply assume that the initial conditions of the universe were such that they lead to the observed baryon to photon ratio.[2]

2 Equilibrium Thermodynamics

Having established that our early universe was in a state of (local) thermal equilibrium, we will now review some basic facts of thermodynamics and then apply them to our universe.

In order to understand the number density n, energy density ρ and pressure P [3] for different particles in the early universe we need to know their distribution as a function of phase space, i.e. their distribution in real space and momentum space. For a homogeneous distribution, this phase space function cannot depend on the spacial coordinate $\vec{r}$ and for an isotropic distribution the phase space function can only depend on the absolute value of the momentum $p = |\vec{p}|$. For a system of particles in equilibrium the distribution function is given by the Fermi-Dirac distribution for fermions and by the Bose-Einstein distribution for bosons. Both can be written as

$$f_\pm(p) = \frac{1}{e^{(E(p)-\mu)/T} \pm 1} \,, \tag{13}$$

[2]There are a variety of theoretical ideas of how such an asymmetry can arise but so far the experiments have not singled out any particular model, so we refrain from discussing baryogenesis in any further detail.

[3]We will switch conventions and denote the pressure by P to avoid confusion with the absolute value of the momentum $p = |\vec{p}|$.

where the + sign is for fermions and the − sign is for bosons and μ denotes the chemical potential.

Due to the ample interactions in the early universe all particles have the same average kinetic energy, i.e. the same temperature T so that we do not need to keep track of different temperatures. In the early universe the chemical potentials for all the particles are small so that we can neglect them and set $\mu = 0$. However, this would mean that the number of particles and anti-particles is the same which isn't quite true as discussed above. A small non-zero chemical potential allows one to account for the small matter-anti-matter asymmetry (c.f. the discussion of Bayogenesis above) and the but it substantially complicates the analysis and is not needed for our discussion.

Allowing for g internal degrees of freedom, i.e. for particles with spin, the particle density in phase space is given by $\frac{g}{(2\pi)^3} f(p)$, where we dropped the subscript $\pm$ to avoid cluttering. In order to obtain the number density n, we need to integrate this over the momentum

$$n \equiv \frac{g}{(2\pi)^3} \int d^3p f(p) \,. \tag{14}$$

To obtain the energy density ρ, we need to weigh each state by its energy $E(p) = \sqrt{m^2 + p^2}$ so that we have [4]

$$\rho \equiv \frac{g}{(2\pi)^3} \int d^3p f(p) E(p) \,. \tag{15}$$

Lastly the pressure is defined as

$$P \equiv \frac{g}{(2\pi)^3} \int d^3p f(p) \frac{p^2}{3E(p)} \,. \tag{16}$$

The integrals in n, ρ and P have to be evaluated numerical unless we are in particular limits. For such limits we will make use of the general formulas

$$\int_0^\infty du \frac{u^n}{e^u - 1} = \zeta(n+1)\Gamma(n+1) \,, \tag{17}$$

$$\int_0^\infty du u^n e^{-u^2} = \frac{1}{2}\Gamma\left(\tfrac{1}{2}(n+1)\right) \,, \tag{18}$$

where ζ is the Riemann zeta-function and the Γ-function is an extension of the factorial function and in particular takes the values $\Gamma(n) = (n-1)!$ for $n \in \mathbb{N}^*$.

The relativistic limit

Let us first evaluate n, ρ and P for relativistic particles:

$$E(p) = \sqrt{m^2 + p^2} \approx p \gg m \,. \tag{19}$$

We define $y = p/T$ so that $f_\pm(y) = 1/(e^y \pm 1)$. For bosons we then find

$$n_b = \frac{g}{(2\pi)^3} \int_0^\infty \frac{4\pi T^3 y^2 dy}{e^y - 1} = \frac{g T^3 \zeta(3)\Gamma(3)}{2\pi^2} = \frac{\zeta(3)}{\pi^2} g\, T^3 \,, \tag{20}$$

[4]For strongly interacting particles we would have to take into account the interaction energy, but the particles in the early universe were weakly interacting so that we can neglect the interaction energy.

where $\zeta(3) \approx 1.2$. For fermions we can use that

$$\frac{1}{e^y+1} = \frac{1}{e^y-1} - \frac{2}{e^{2y}-1}\,, \tag{21}$$

to get

$$n_f = n_b - \frac{g}{(2\pi)^3}\int_0^\infty \frac{8\pi T^3 y^2 dy}{e^{2y}-1} = n_b - \frac{g}{(2\pi)^3}\int_0^\infty \frac{\pi T^3 \tilde{y}^2 d\tilde{y}}{e^{\tilde{y}}-1} = n_b - \frac{1}{4}n_b = \frac{3\zeta(3)}{4\pi^2} g\,T^3\,. \tag{22}$$

So we have found for relativistic particles that

$$n_b = \frac{4}{3}n_f = \frac{\zeta(3)}{\pi^2} g\,T^3\,. \tag{23}$$

Note, that the scaling of T^3 agrees with our scaling assumption in equation (5).