Cosmology and particle physics

Lecture notes

Timm Wrase

Lecture 7 The thermal universe - part III

Last time we have seen that our universe roughly $10^{-12}s$ after the big bang was filled with a 'soup' of standard model particles that are all in thermal equilibrium. We described how during the expansion of our universe, due to the decrease in temperature, particles become non-relativistic. Today we start at an energy of roughly $1MeV$, which correspond to $1s$ after the big bang, and we will discuss the fate of electrons, positrons, photons and neutrinos. In order to do that it is useful to keep track of a conserved quantity, namely the entropy.

1 The entropy of our universe

In the second homework you used the first law of thermodynamics

$$TdS = dE + PdV\,, \tag{1}$$

to derive the continuity equation $\dot{\rho}+3H(\rho+P)=0$ under the assumption that the entropy is not changing in an expanding universe. The continuity equation follows from the Friedmann equations that we use to describe our universe which means that the entropy in our universe is conserved. This means it is a useful quantity to keep track of.

To do so we recall that the entropy and the energy are extensive quantities which satisfy

$$\partial_V E = \frac{E}{V}\,, \qquad \partial_V S = \frac{S}{V}\,. \tag{2}$$

Using this and $S = S(V,T)$ and $E(V,T)$ we find from equation (1)

$$\begin{aligned} T\partial_T S dT + T\partial_V S dV &= \partial_V E dV + \partial_T E dT + PdV \\ T\partial_T S dT + T\frac{S}{V}dV &= \frac{E}{V}dV + \partial_T E dT + PdV\,. \end{aligned} \tag{3}$$

Since dT and dV are independent, in particular the terms multiplying dV have to vanish, which gives

$$T\frac{S}{V} = \frac{E}{V} + P \quad \Rightarrow \quad S = \frac{E+PV}{T}\,. \tag{4}$$

We now define the entropy density

$$s \equiv \frac{S}{V} = \frac{\rho+P}{T}\,. \tag{5}$$

For our discussion of the early universe we are interested in radiation with $P=\frac{1}{3}\rho$ for which the entropy density becomes

$$s = \frac{4}{3}\frac{\rho}{T}\,. \tag{6}$$

Note, that it easily follows from the scalings $\rho \sim a^{-4}$, $V \sim a^3$ and $T \sim a^{-1}$ that the entropy S is indeed constant.

In order to calculate the entropy density in the early universe, we have to sum over all particles, taking into account their energy density and temperature. We do this like last time by defining

$$s = \frac{2\pi^2}{45} g_{\star S}(T) T^3 \,, \tag{7}$$

where we used equation (14) from the lecture 6 notes for the energy density ρ and T denotes the photon temperature. The quantity $g_{\star S}(T)$ again is the sum of the particles in thermal equilibrium with the photons and the decoupled particles $g_{\star S}(T) = g_{\star S}^{th} + g_{\star S}^{dec}(T)$. The effective number in thermal equilibrium is the same as last time $g_{\star S}^{th} = g_{\star}^{th}$, where $g_{\star}^{th}$ was defined in equation (23) in the lecture 6 notes. However, due to the different scaling with temperature, this is not true for the decoupled degrees of freedom. In particular, we have

$$g_{\star S}^{dec}(T) = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^3 \,, \tag{8}$$

which is different from $g_{\star}^{dec}(T)$ which has powers of 4 for the temperature (cf. equation (24) in the lecture 6 notes).

As we showed above, the entropy is preserved and in the case that no particle are created or destroyed then the particle density and entropy density scale in the same way, i.e. like $1/V \sim a^{-3}$. This means that their ratio n_i/s is constant. Now in the early universe in thermal equilibrium particles and anti-particles are constantly created and destroyed but as we discussed before the net baryon number cannot change due to perturbative standard model interactions. So after baryogenesis the quantity $(n_b - n_{\bar{b}})/s$ is preserved and does not change during the evolution of our universe until today.

Another consequence of the fact that $sa^3 = const.$ is that, using equation (7), we find that

$$g_{\star S}(T) T^3 a^3 = const. \quad \Rightarrow \quad T \propto g_{\star S}^{-\frac{1}{3}} a^{-1} \,. \tag{9}$$

This means that away from temperatures where particles become non-relativistic we find that the factor of proportionality, i.e. the slope of the decrease of the temperature, is constant and depends on the relativistic degrees of freedom. If one species drops out of equilibrium because it becomes non-relativistic, then its entropy density (like its energy density) decays exponentially. However, the net entropy has to stay constant so the particle that becomes non-relativistic has to transfer its entropy to the particle species that are still in thermal equilibrium. For example, when electrons and positions are in thermal equilibrium we have the reaction

$$e^- + e^+ \leftrightarrow \gamma + \gamma \,. \tag{10}$$

Once the temperature drops below the electron mass, the annihilation is strongly favored

$$e^- + e^+ \rightarrow \gamma + \gamma \,, \tag{11}$$

the electron and positron densities decay exponentially and their entropy is transfered to the photons. Since $g_{\star S}$ decreased by $\frac{7}{8}4$ during this decoupling of the electrons, the factor of proportionality between T and a^{-1} in equation (9) has increased. This is shown in figure 1.

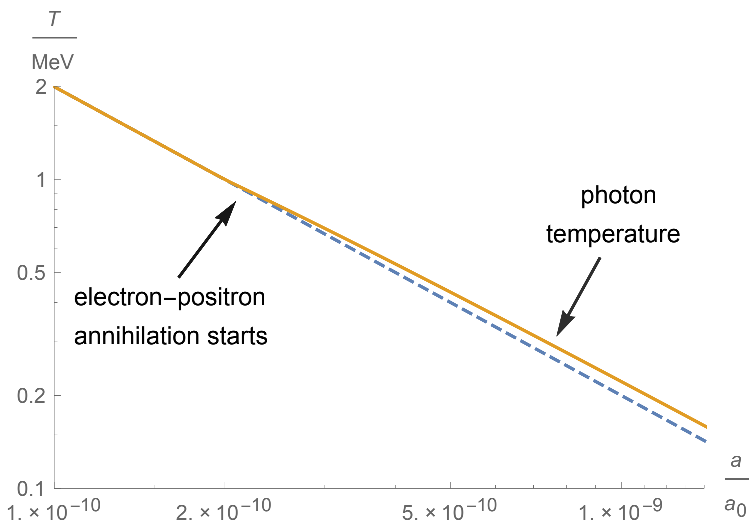

Figure 1: After the electrons and positrons become non-relativistic they annihilate which changes the evolution of the temperature of the photons (orange line).

2 Neutrino decoupling

In our universe things are more interesting and in particular the neutrinos play an interesting role. Via weak interactions like for example

$$e^- + e^+ \leftrightarrow \nu_e + \bar{\nu}_e \,, \tag{12}$$

they are kept in thermal equilibrium with the electrons. We have argued based on dimensional analysis that the cross-section for such interactions goes like $\sigma \sim \alpha^2/T^2$. However, this is not anymore true after the electroweak symmetry breaking. After this electroweak symmetry breaking the $W^\pm$ and Z bosons have a mass which provides a new scale and actually determines the weak interaction strength. A process with two initial and two final particles like the one in equation (12) scales like $\alpha^2/m_W^4 \approx 10^{-10} GeV^{-4}$. On dimensional grounds we then have

$$\sigma \sim \frac{\alpha^2}{m_W^4} T^2 \,, \tag{13}$$

so instead of increasing with decreasing temperature, σ is now decreasing during the cooling of the universe.

Let us ask what this means for the ratio between the particle interaction rate $\Gamma = n\sigma v$ compared to the Hubble expansion rate $H \sim T^2/M_P$. Recalling that $n \sim T^3$ and $v \approx 1$, we get

$$\frac{\Gamma}{H} \sim \frac{\alpha^2}{m_W^4} T^5 \frac{M_P}{T^2} \approx \left(\frac{T}{MeV}\right)^3 . \tag{14}$$

What this estimate shows is that particles like the neutrinos that only interact via the weak force, will freeze-out at a temperature of roughly $1 MeV$.

3 The cosmic neutrino background

The only particles that are relativistic at such low energies are the electron, the photon and the neutrinos. The electron becomes non-relativistic at slightly lower temperature $T \lesssim m_e = .5MeV$. At this point the neutrinos are decoupled and the entropy of the electrons is transfered only to the thermal bath of the photons. This means that the neutrinos will have a lower temperature than the photons. In figure 1, the neutrino temperature would follow the dashed blue line, while the photon temperature corresponds to the orange line.

Let us make this more precise: Neglecting the decoupled neutrinos, we have before and after the electron decoupling

$$g_{\star S} = \begin{cases} 2 + \frac{7}{8}4 = \frac{11}{2} & T > m_e \\ 2 & T \ll m_e \end{cases} . \tag{15}$$

It then follows from equation (9) that the factor of proportionality between the photon temperature T_γ and a^{-1} changes due to the electron decoupling by a factor of $((11/2)/2)^{1/3} = (11/4)^{1/3}$. However, the same factor for the neutrinos does not change so that the neutrino temperature is slightly lower and given by

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \approx .71\, T_\gamma\,. \tag{16}$$

This means that our universe is filled in addition to the cosmic microwave background with a cosmic neutrino background that currently has a temperature of

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} 2.725K \approx 1.95K\,. \tag{17}$$

We are currently not able to experimentally detect this cosmic neutrino background since the neutrinos have very low energies and interact only via the weak force. However, we have indirect evidence for their existence since their energy and entropy density affect the big bang nucleosynthesis and anisotropies in the cosmic microwave background, both of which we will discuss later in this course.

Next let us calculate the values of $g_\star$ and $g_{\star S}$ after the decoupling of the electrons. For $g_\star$ we have from equations (23)-(25) in the lecture 6 notes

$$g_\star(T) = g_\star^{th} + g_\star^{dec}(T) = 2 + \frac{7}{8} \cdot 2 \cdot 3 \cdot \left(\frac{4}{11}\right)^{\frac{4}{3}} \approx 3.36\,. \tag{18}$$

Similarly, we find for $g_{\star S}$ from equation (8)

$$g_{\star S}(T) = g_{\star S}^{th} + g_{\star S}^{dec}(T) = 2 + \frac{7}{8} \cdot 2 \cdot 3 \cdot \frac{4}{11} \approx 3.91\,. \tag{19}$$

These are the values shown in figure 2, that we have already seen last time.[1] As is also clear from figure 2, before the neutrino freeze-out we have $g_\star = g_{\star S}$.

[1] The decoupling of the neutrinos is not instantaneous and not totally finished by the time the electrons become non-relativistic. This means that a small fraction of the electron entropy is transferred to the neutrinos. This is usually encoded in an effective number of neutrinos $N_{eff} \approx 3.046$ that replaces the 3 that counts the neutrinos in equations (18), (19) which leads to the values in figure 2.

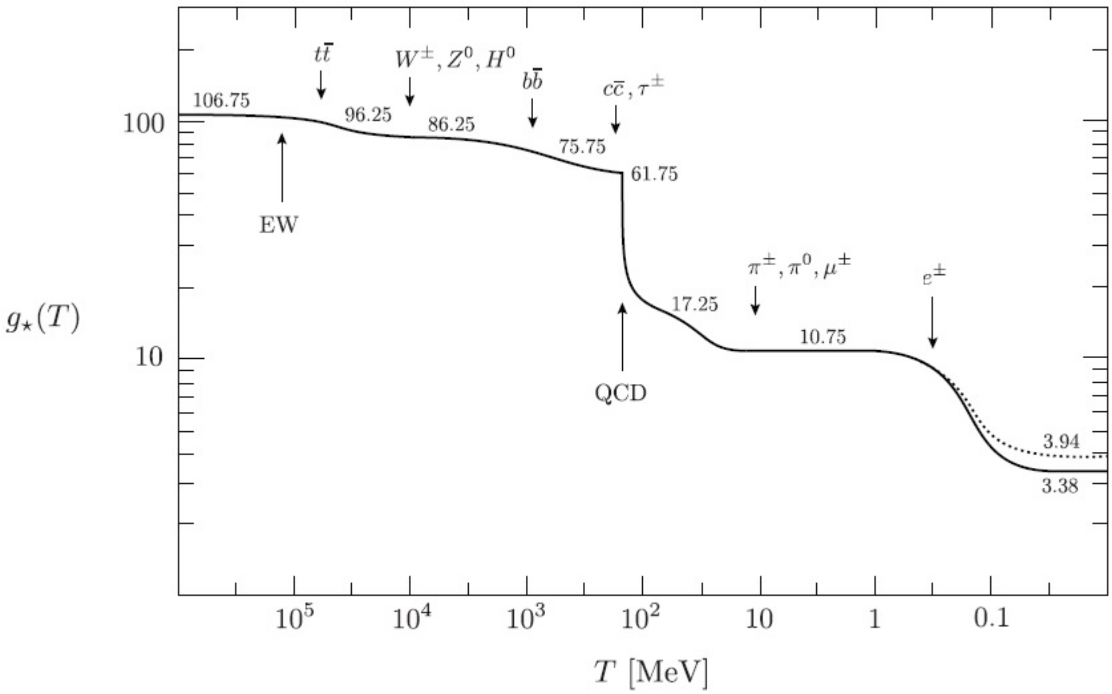

Figure 2: The evolution of the relativistic degrees of freedom in our early universe (taken from Daniel Baumann's "Cosmology" lectures).

4 Important events in our early universe

Before we start to discuss the deviation from equilibrium and big bang nucleosynthesis next time, it might be good to take stock and recall the important events of our universe in the time from $10^{-12}s$ to $380,000$ years after the big bang.

As we have discussed before the time of $10^{-12}s$ after the big bang corresponds to a temperature of $1TeV$ which is roughly the energy scale we can test in particle experiments like the LHC at CERN. We have also argued that the standard model particles are in thermal equilibrium at that time so that we have a hot and dense soup of standard model particles at this point. The expansion of the universe leads to a cooling of this plasma and the important events during the evolution and cooling of our universe are summarized in the table below.

Event	Time	Redshift	Temperature
Baryogenesis	?	?	?
EW phase transition	$2 \times 10^{-11} s$	10^{15}	$100 GeV$
QCD phase transition	$2 \times 10^{-5} s$	10^{12}	$150 MeV$
Neutrino decoupling	$1s$	6×10^9	$1 MeV$
Electron-positron annihilation	$6s$	2×10^9	$500 keV$
Big bang nucleosynthesis	$3min$	4×10^8	$100 keV$
Matter-radiation equality	$6 \times 10^4 yrs$	3400	$.75 eV$
Recombination	$2.6 - 3.8 \times 10^5 yrs$	1100-1400	$.26 - .33 eV$
CMB	$3.8 \times 10^5 yrs$	1100	$.26 eV$

Baryogenesis: As we discussed in lecture 5, there is an asymmetry between baryons and anti-baryons that cannot be explained by the standard model of particle physics. Thus at energies above $1 TeV$ there must be some new physics that generates this asymmetry. While there are many different theoretical ideas, there is no experimental test of any of these so we cannot associate a time to baryogenesis. Since the observed universe is neutral under the electric charge, there must be a similar asymmetry between electrons and positrons so that after their annihilation we are left with one electron for each proton.

Electroweak-phase transition: During this phase transition that we discussed last time, the particles get their mass due to the so called Higgs effect. Once the standard model particles are massive they start to drop out of equilibrium whenever the temperature of the universe (i.e. the thermal bath) becomes smaller than their mass. Then the particles start to annihilate with their anti-particles and their number densities decrease exponentially. The remaining matter in our observed universe is due to the matter-anti-matter asymmetry mentioned above.

QCD phase transition: The strong force is weaker at higher energies (temperatures) and becomes stronger and stronger during the cooling of the universe. Around $150 MeV$ the strong force is so strong that free gluons and quarks cannot exist anymore and all the quarks are bound into so called baryons and mesons. These are bound states that are neutral under the strong force. The lightest baryons are the familiar proton and neutron. There are also heavier baryons and mesons that can be lighter than the proton and neutron but all of these are unstable and quickly decay. So a little bit after the QCD phase transition we are left with essentially only protons and neutrons that are the building blocks for the atomic nuclei.

Neutrino decoupling: As we discussed today around $1 MeV$ the weak interaction becomes so weak that particles that are only charged under the weak force, i.e. the neutrinos, decouple from the thermal plasma. These neutrinos, similarly to the photons in the CMB, give rise to a cosmic neutrino background that is slightly colder than the CMB and is difficult to observe directly. At the time of decoupling the three neutrinos are still relativistic and during the cooling of the universe they become non-relativistic whenever the temperature is smaller than their respective mass.

Electron-positron annihilation: Around $T \sim m_e \approx 511keV$ the electrons and positrons become non-relativistic and transfer their energy and entropy into the photons only (since the neutrinos are decoupled already). This slows down the decrease in the temperature of the photons a little bit so that the photons today have a temperature that is a little bit larger than the neutrino background.

Big bang nucleosynthesis: One of the greatest successes of the big bang cosmology is that it correctly predicts the observed abundance of elements in our universe. We will discuss next time how the protons and neutrons in our universe combine into atomic nuclei. Using nuclear physics, we can predict the amount of different elements in the early universe and these predictions agree with what we observe. Any kind of new physics that can appear beyond the standard model is severely constraint by this success.

Recombination: Once the average energy of the photons drops below $.33eV$ the tail of high energy photons is sufficiently small to allow for neutral atoms to form. This process in which electrons and protons combine[2] takes roughly 100,000 years and at its end the universe is filled with clouds of neutral atoms and the cosmic microwave background.

The cosmic microwave background: Once the electrons and nuclei combine into neutral atoms, the photons can stream freely until today. The observation of this cosmic microwave background does not only tell us about the universe 380,000 years after the big bang but the incredible homogeneity of the CMB also strongly motivates a short phase of accelerated expansion in our very early universe, the so called inflation. The small deviations from homogeneity in the CMB photons we observe together with their polarization provide detailed information about this period of inflation.

5 Dark Matter

As we have seen in lecture 3, more than 80% of the matter in our universe is not in the form of standard model particles but rather in the form of dark matter that consists of one (or multiple) unknown particle species. Since we don't know what these particles are and how they interact, we cannot say for sure how their observed energy density arises. However, if we assume that the dark matter and standard model particles are in thermal equilibrium in the early universe, then the evolution of the dark matter particles should be describable with the tools we developed so far. In particular, since the dark matter is not visible and so difficult to detect, it can at most interact with the standard model particles via the weak force (or via an unknown even weaker force). If we assume that the mass of the dark matter

[2] Don't ask why this is called *re*combination. The electrons and nuclei where never combined before that point.

particles is above their decoupling scale, then they would become non-relativistic before they decouple from the standard model particles and their number density starts to exponentially decay. Then at a certain point their interaction rate becomes so small that $\Gamma \sim H$ which leads to a freeze-out and a relic density of dark matter particles. This scenario can lead to the observed amount of dark matter and experiments have already substantially constraint the cross-section and mass of such weakly interacting massive particles (often called WIMPs). This is shown in figure 3 where the constraints on the WIMP mass is plotted against the WIMP-nucleon cross section. The upper right region is excluded by experiments.

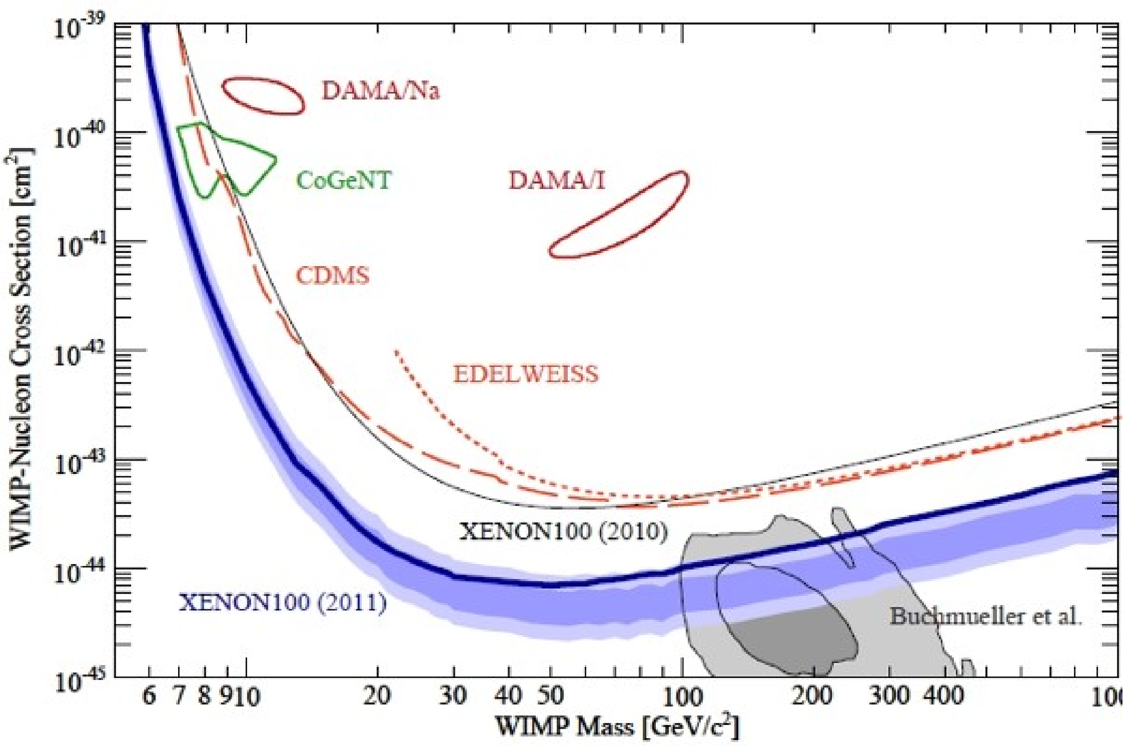

Figure 3: Some experimental bounds on weakly interacting massive particles.

While some experiments have claimed a detection of dark matter in the past, other experiments could not reproduce these findings. So we have to wait for future experiments to shed light on the nature of the dark matter in our universe.