Cosmology and particle physics

Lecture notes

Timm Wrase

Lecture 9 Inflation - part I

Having discussed the thermal history of our universe and in particular its evolution at times larger than 10^{-12} seconds after the big bang, we will now venture even closer to the initial singularity and discuss the theory of inflation. We will first layout the original problems that cosmologists were facing before Guth, Linde, Albrecht and Steinhardt invented inflation in the 1980's. Then we explain how inflation solves these problems.

1 Beyond ΛCDM

To describe the evolution of our universe so far we have been able to use well tested particle and nuclear physics and two extra ingredients: the cosmological constant Λ and cold dark matter (CDM), where cold refers to the fact that this matter behaves like non-relativistic matter with equation of state parameter $w = 0$. This so called ΛCDM model seems to correctly describe the evolution of our universe from 10^{-12} seconds after the big bang until today. However, we have already seen that there has to be something else that we don't understand yet: The asymmetry between matter and anti-matter requires processes in the earlier universe that go beyond the standard model of particle physics. In addition it seems that our universe underwent a period of inflation during very early times. While experiments provide us with ever improving bounds on different inflationary models, they have not yet singled out one particular model of inflation so we will discuss a variety of different models and their features. Since the theory of inflation is less well tested, we should ask what its generic predictions are and which of those we observe. As discussed above, cosmologists up until 1980 were faced with some problems that get resolved, if our very early universe underwent a period of inflation. However, the absence of these problems is then not a prediction but rather a post-diction of inflation since it was invented to resolve these issues. Does inflation make in addition any generic predictions that we can test? Yes, as we discuss at the end of this course, all inflationary models predict a nearly scale invariant spectrum for the density perturbation that are tiny deviations from an entirely homogeneous universe. The imprint of these density perturbations has been observed in the cosmic microwave background. So it is fair to say that the observational evidences for the theory inflation are pretty robust and our universe most likely underwent such a phase at some time around maybe 10^{-34} seconds after the big bang.

1.1 The horizon problem

The first problem in an early universe that is only dominated by radiation and matter is called the horizon problem. Recall that the photons of the cosmic microwave background decoupled 380,000 years after the big bang and they constitute the best black body spectrum ever observed in nature. This black body spectrum has the same temperature everywhere in the sky. In particular this means that all the photons that come to us from one side have the same temperature as the photons that come from the opposite side. This seems only possible, if these photons have been in causal contact with each other so that they can be in thermal equilibrium. We have previously discussed that there are abundant interactions in the early universe that establish a local equilibrium but we haven't discussed the size of these local patches in equilibrium. In order to so, it is useful to work again with conformal time τ.

Recall from the beginning of lecture 4 that the metric in conformal times is given by

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + \frac{dr^2}{1 - Kr^2} + r^2 d\Omega \right) , \tag{1}$$

and a radially traveling light ray satisfies

$$ds = 0 \qquad \Rightarrow \qquad d\tau = \frac{dr}{\sqrt{1 - Kr^2}} \equiv d\chi \, . \tag{2}$$

In particular this means that in the (χ, τ)-plane light rays travel along straight lines at 45° angles. For each point P in the (χ, τ)-plane we can draw a future light-cone and a past light-cone by drawing two straight lines at ±45° angles that intersect in P. Every point in spacetime inside the past light-cone can send information to the point P and every point in future light-cone can receive information from the point P. We have already discussed these kind of causally connected parts in lecture 4. The radius of the future light-cone is the event horizon and the radius of the past light-cone is the particle horizon (see figure 1).

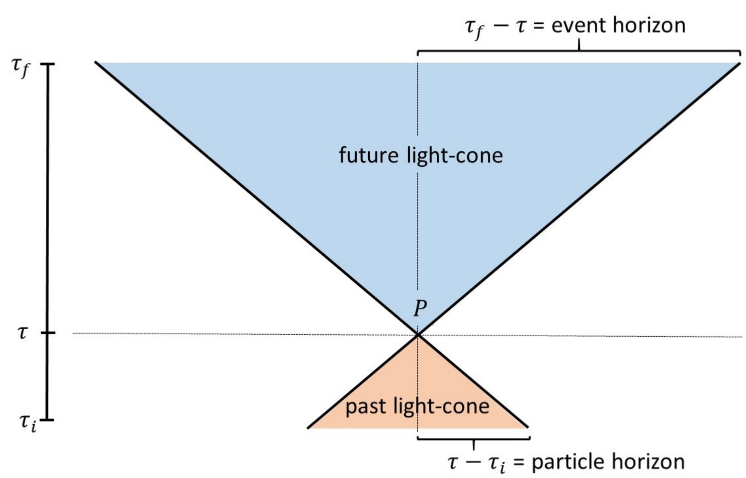

Figure 1: The future and past light-cone associated to the point P.

The question we have to ask is whether all the points on the surface of last scattering that we observe today have been in causal contact or not? The answer is no! This means that in an early universe that is radiation dominated the photons that are coming from one side of the universe and are reaching us today have never been in causal contact with the photons coming from the other side of the universe. So why are they both having the same temperature? This is the horizon problem and is depicted in figure 2.

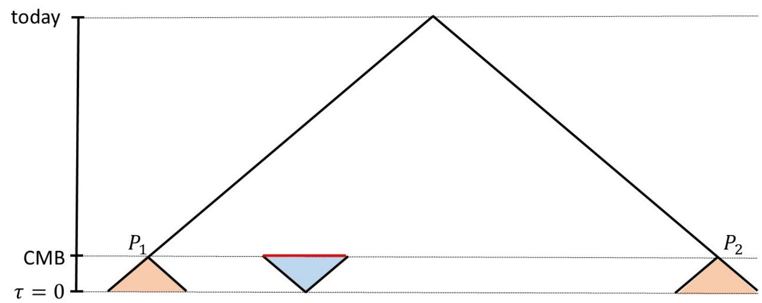

Figure 2: The point P_1 and P_2 have past light-cones that do not intersect so these two points have never been in causal contact. The blue cone is the future light-cone of one point on the initial singular surface and the red line shows the maximal surface size that could have been in causal contact.

The initial singularity with $a(\tau = 0) = 0$ has non-zero (and maybe even infinite) comoving spatial size. If we assume that the points on this initial surface are not particularly fine tuned to have all the same initial conditions then we expect thermal equilibrium to lead to the same temperature only for rather small patches of the sky. A simple calculation for our universe reveals that only photons coming from within a $2°$ angle should be in thermal equilibrium. This means that the surface of last scattering should consist of roughly $4\pi/(2° * 2\pi/360°)^2 \approx 10^5$ different patches that have not been in thermal equilibrium.

We will see later that there are actually small fluctuations in the temperature of the CMB of the order of .01%. While this is incredibly small, it raises the question of how different causally disconnected patches would have to be in order not to worry about the fine tuning of the initial surface. I have no good answer to this but once we know about these small fluctuations in the CMB then there is an actually well posed horizon problem: The fluctuation in the temperature are correlated on scales much larger than $2°$, so how is this possible if these regions have not been in causal contact?

1.2 The flatness problem

The second problem is also related to fine tuning. Recall from lecture 3 that the curvature contribution to the energy density of our universe is very small. This means that the normalized total energy density today is $\Omega_T \approx 1$. More specifically from equation (4) and (2) in the lecture 3 notes we have

$$\Omega_T - 1 = \frac{K}{\dot{a}(t)^2} = \frac{3K}{8\pi G\, \rho_c(t)\, a(t)^2}\,. \tag{3}$$

Today the experimental bounds is

$$\frac{K}{\dot{a}(t_0)^2} < .005\,. \tag{4}$$

Let us look at the last term in equation (3). If the energy density is dominated by a cosmological constant then $\rho_c(t)$ is constant, however, during a matter dominated era we have $\rho_c(t) \propto a(t)^{-3}$ and during the radiation dominated era we have $\rho_c(t) \propto a(t)^{-4}$. This means that any small deviation of Ω_T from one will grow during the radiation and matter dominated era. Likewise, going back in time we find that the value of Ω_T must have been incredibly close to 1 in the early universe. At matter-radiation equality ($z = 3400$) we have roughly

$$\Omega_T - 1 < .005 \frac{1}{3400} \approx 1.5 \times 10^{-6}\,, \tag{5}$$

and at the time of the electro-weak phase transition with $T \approx 100 GeV$ and $z = 10^{15}$ we have

$$\Omega_T - 1 < .005 \frac{1}{3400} \left(\frac{3400}{10^{15}}\right)^2 \approx 1.7 \times 10^{-29}\,. \tag{6}$$

Finally, at the Planck scale $T \approx M_P = 2.2 \times 10^{18} GeV$ we would have

$$\Omega_T - 1 < .005 \frac{1}{3400} \left(\frac{3400}{10^{15} \cdot 2.2 \times 10^{16}}\right)^2 \approx 3.5 \times 10^{-62}\,. \tag{7}$$

This seems like an incredible fine tuning, which could either be explained by $K = 0$ or requires very special initial conditions. Since we know of no reason why $K = 0$ in our universe, we are again faced with very finely tuned initial conditions.

1.3 The monopole problem

Alan Guth was thinking about so called grand unified theories (GUT) in which all forces of the standard model of particle physics are unified in a single force around an energy scale of approximately $10^{16} GeV$. This single forces is broken into the strong, weak and electromagnetic force at energies below $10^{16} GeV$ and usually this phase transition leads to unwanted relics like for example magnetic monopoles. The existence of such heavy particles that would be produced during the phase transition can overclose the universe ($\Omega \gg 1$). So Guth original motivation for inflation was to remove the overabundance of these heavy relics in GUT theories.

2 Inflation

Inflation is a period of accelerated expansion of our universe that happened at very early times. Here we will focus on the case in which the universe is approximately exponentially expanding as is the case during an era that is dominated by a cosmological constant. Such a period solves the three problems above, if it lasts sufficiently long. We can make this more precise and determine a minimal amount of exponential expansion that is required to solve each of the problems above.

2.1 Solving the horizon problem

If in the very early universe there would have been a phase during which $a(t)$ grows exponentially, then a small patch in local thermal equilibrium could be stretched to the size of the surface of last scattering that we observe today and thereby solve the horizon problem.[1] More precisely, if the early universe would have been in a phase with $a(t) \propto e^{Ht}$, then the beginning of the universe would not be at $t = 0$ anymore but at $t = -\infty$. This would likewise push the initial conformal time τ_i to $-\infty$, while any finite period of inflation pushes τ_i to a negative but finite value. This can then allow for causal contact of all the points we observe on the surface of last scattering as is shown in figure 3.

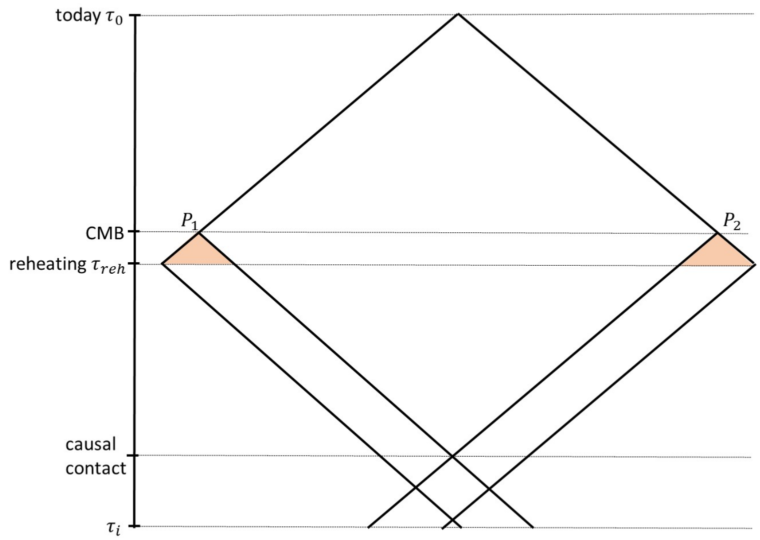

Figure 3: The point P_1 and P_2 have past light-cones that can intersect, if we have a period of exponential expansion that pushes τ_i sufficiently far back.

Before we discuss how such a period of inflation can arise in our very early universe, we would like to get an estimate for how long it would have to last in order to solve the horizon problem. From the diagram 3, it is clear that need the conformal time between the beginning of inflation τ_i and the end of inflation which is around the reheating time τ_{reh} to be at least as large as the time between reheating and today at τ_0.

Recalling that $d\tau \equiv a(t)^{-1}dt$ we get

$$\tau_{reh} - \tau_i = \int_{\tau_i}^{\tau_{reh}} d\tau' = \int \frac{dt'}{a(t')} = \int \frac{da}{a\dot{a}} = \int_{a_i}^{a_{reh}} \frac{da}{a^2 H_{inf}} \approx \frac{1}{a_i H_{inf}} - \frac{1}{a_{reh} H_{inf}} \approx \frac{1}{a_i H_{inf}}\,, \quad (8)$$

where we used that the Hubble constant during inflation is approximately constant and that $a_i \ll a_{reh}$ due to the exponential expansion during inflation.

To get a very simple (but fairly accurate estimate) we assume that the universe after the end of inflation is radiation dominated so that $a(t) = a_0\sqrt{t/t_0}$ and $a^2 H = const. \approx a_{reh}^2 H_{inf}$ since $a(t)$ is a smooth function. This then leads to

$$\tau_0 - \tau_{reh} = \int_{\tau_{reh}}^{\tau_0} d\tau' = \int_{a_{reh}}^{a_0} \frac{da}{a^2 H} \approx \frac{1}{a_{reh}^2 H_{inf}}(a_0 - a_{reh}) \approx \frac{a_0}{a_{reh}^2 H_{inf}}\,, \quad (9)$$

[1] Actually things are more complicated: Inflation itself needs somewhat special initial conditions to start, so that the horizon problem is substantially alleviated but not completely solved. After inflation ends the temperature of the universe is essentially zero, because $T \propto a(t)^{-1}$ and $a(t)$ grows exponentially during inflation. However, the inflaton field carries energy that is then used to homogeneously reheat the universe.

where we used that $a_0 \gg a_{reh}$. We then find the constraint

$$\begin{aligned}
\tau_{reh} - \tau_i &\gtrsim \tau_0 - \tau_{reh} \\
\frac{1}{a_i H_{inf}} &\gtrsim \frac{a_0}{a_{reh}^2 H_{inf}} \\
\frac{a_{reh}}{a_i} &\gtrsim \frac{a_0}{a_{reh}} \approx \frac{T_{reh}}{T_0}\,,
\end{aligned} \tag{10}$$

where we used in the last line that the temperature scales like the inverse of the scale factor. For example fo inflation with an energy scale of $T_{reh} \approx 10^{14} GeV$ we then find

$$\frac{a_{reh}}{a_i} \gtrsim 10^{26} \approx e^{60} \equiv e^{N_e}\,. \tag{11}$$

We see that the expansion factor during inflation is gigantic so that one defines the number of e-folds N_e that is given by the logarithm to basis e of the expansion factor. The number of e-folds required to solve the horizon problem depends on the energy scale that we chose to be $10^{14} GeV$ above and that is not known. However, the standard values in the literature are $N_e = 50 - 60$ which nicely matches with the value derived above.

2.2 Solving the flatness problem

The flatness problem is solved because during inflation $a(t) \propto \dot{a}(t)$. The large grows of $\dot{a}(t)$ then suppresses the term $\frac{K}{\dot{a}(t)^2}$ in equation (3). If we would take $\frac{K}{\dot{a}(t)^2}$ to be initially some order one number and we want it to be sufficiently small at the end of inflation to explain the observed value, then we need for example for inflation ending slightly below the GUT scale around $10^{14} GeV$ that

$$\frac{K}{\dot{a}(t_i)^2} \approx 1\,, \tag{12}$$

$$\Omega_T(t_{reh}) - 1 = \frac{K}{\dot{a}(t_{reh})^2} \lesssim .005\frac{1}{3400}\left(\frac{3400}{10^{15}\cdot 10^{12}}\right)^2 \approx 1.7\times 10^{-53}\,, \tag{13}$$

$$\Rightarrow \frac{\dot{a}(t_{reh})}{\dot{a}(t_i)} \approx \frac{a(t_{reh})}{a(t_i)} \gtrsim \left(1.7\times 10^{-53}\right)^{-\frac{1}{2}} \approx e^{60}\,, \tag{14}$$

where we used that during inflation $a(t) \approx e^{H_{inf} t}$ with H_{inf} the constant Hubble parameter during inflation. We see that we again need roughly 60 e-folds of inflation to solve the flatness problem and this value is of course related to the energy scale at which we would like to solve the flatness problem. If for example we wanted to solve the flatness problem at an energy of $100 TeV$, then we would only need 40 e-folds of inflation.

2.3 Solving the monopole problem

It is intuitively clear that a period of inflation will also solve the monopole problem, provided that the monopoles and other relics are not produced after inflation. If magnetic monopoles would be produced at the GUT scale, then a period of inflation that takes places at lower energies dilutes away all the relics and leaves us with an almost empty universe. The amount of inflation necessary to solve the monopole problem is usually a little bit lower and roughly 30 e-folds are usually enough to sufficiently dilute the relics so that they don't have any impact on the cosmological evolution and wouldn't be observable today. However, if there is a GUT theory at energies around $10^{16}GeV$ then the reheating temperature after inflation has to be lower than this $10^{16}GeV$. The current upper bound on the energy scale during inflation is around the GUT scale so that the reheating temperature is also bounded from above by the GUT scale and there is no tension between grand unified theories and inflation.

3 A period of inflation from a scalar field

A great problem with an early period in our universe that is dominated by a large cosmological constant is that the energy density of the cosmological constant does not decay during the expansion, while it does so for matter and radiation. This means, as we discussed previously, that a large cosmological constant is inconsistent with our observed universe. So what we need for inflation is a fluid that mimics a large cosmological constant for a short period of time and then transfers its energy into the other particles of our universe during a reheating process and afterwards essentially disappears. This can be accomplished by a scalar field, i.e. a spinless particle similar to the Higgs field (but most likely not the standard model Higgs particle).

The action for such a scalar field ϕ, that is called the inflaton, is given by

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) . \tag{15}$$

Here $V(\phi)$ is the potential for the scalar field that we leave arbitrary. You can think of this scalar field as a 'ball' rolling in a potential. The only difference here is that ϕ in principle depends not only on the time t but also on the spatial coordinates x^i so the value of ϕ can change throughout space. You are probably familiar with this from electrodynamics where the electric and magnetic fields can vary through space and time.

As is explained in the handout, the variation of the above action with respect to the metric leads to the following energy density and pressure for a scalar field in the FRW universe

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi) , \tag{16}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} - V(\phi) . \tag{17}$$

Note, that if the spatial variation $\nabla\phi$ and the time variations $\dot{\phi}$ vanish, then we have $\rho_\phi = V(\phi) = -P_\phi$ so that the scalar field behaves exactly like a cosmological constant! Guth original idea was that the scalar field sits at a false minimum of the potential as shown in figure 4. The scalar field will then lead to an effective large cosmological constant and a period of inflation. After quantum tunneling through the barrier the scalar field will role down the potential to the true minimum and again lead to an effective cosmological constant that could be the observed value of the cosmological constant today if $V_{today} \approx 10^{-120}M_P^4$.

The problem with Guth original proposal is that the quantum tunneling will happen via the nucleation of spatial bubbles. Inside these bubbles the field is on its way to the true

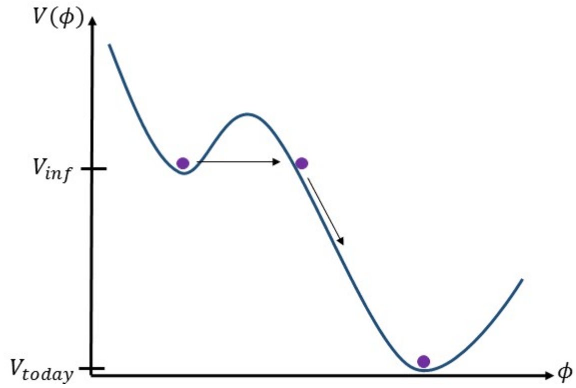

Figure 4: The scalar fields is initially trapped in a false vacuum leading to a period of inflation with a large cosmological constant proportional to V_{inf}. After quantum tunneling the scalar field roles to the true minimum at which the value of the potential is tiny $V_{today} \sim 10^{-120} M_P^4$.

vacuum and outside of the bubbles is the false vacuum. Since the energy is smaller inside of these bubbles, the bubble walls will expand outwards. However, it turned out that these bubbles cannot be large enough to contain our entire universe and the collision of multiple bubbles would lead to inhomogeneities that are larger than the ones we observe. So there is no nice way of ending inflation in this case.

However, shortly after Guth's original idea, Linde, Albrecht and Steinhardt proposed another kind of inflation in which the scalar field is not trapped in a false vacuum but simply rolling very slowly since the scalar potential is very flat as is shown in figure 5. When the inflaton reaches a steeper part of the potential, its kinetic energy becomes important and it does not behave like a cosmological constant anymore and inflation ends. When the scalar fields reaches its true minimum, it will oscillate and couplings to the standard model particles can transfer the kinetic energy of the inflaton into standard model particles which leads to a reheating of the universe and the start of our hot universe that we can describe so well with thermodynamics.

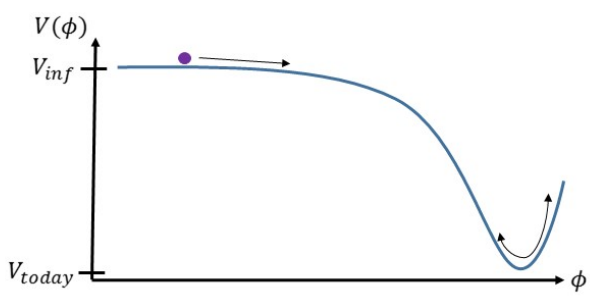

Figure 5: The potential is very flat so that the scalar field roles very slowly leading to a period of inflation with a large cosmological constant proportional to V_{inf}. Once the potential steepens, inflation ends and the scalar field roles to its true minimum with $V_{today} \sim 10^{-120} M_P^4$.