Name: DUE ON 4.4.2016

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Homework - 1

1. A simple toy universe: Let us solve the Friedmann equations in the presence of only a cosmological constant:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a^2} = \frac{\Lambda}{3}, \tag{1}$$

$$\left[\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3}\right]. \tag{2}$$

It is sufficient to solve the first equation since the second one is implied by the first.

(a) Set $K = 0$ and assume that $\Lambda > 0$. Determine $a(t)$ in an expanding universe. On dimensional grounds the cosmological constant should be $\Lambda = \lambda \cdot \frac{c^5}{\hbar G} \approx \lambda \cdot 10^{86}/s^2$. We don't know of any reason why λ should be particularly small so let us set $\lambda = 1$. Assume two atoms are initially separated by a distance of $d_0 = 10^{-10}m$. How long does it take in this expanding universe until their distance equals the distance between the earth and the center of our galaxy? You just discovered the cosmological constant problem. In our universe λ turns out to be incredible small $\lambda \approx 10^{-120}$.

(b) Solve the Friedmann equations for $K = -1$ and a negative cosmological constant. Fix the initial condition so that $a(t = 0) = 0$. Sketch $a(t)$, remembering that the scale factor is determining distances and therefore has to satisfy $a(t) \geq 0$. The fate of such a universe is often called a big crunch.

2. Following the handout "Deriving the Friedmann equations from general relativity" let us calculate some Christoffel symbols. We take for simplicity $K = 0$ which leads to the following metric and four vector

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t)^2 & 0 & 0 \\ 0 & 0 & a(t)^2 & 0 \\ 0 & 0 & 0 & a(t)^2 \end{pmatrix}, \qquad x^\mu = \begin{pmatrix} t \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}. \tag{3}$$

(a) Calculate the inverse metric $g^{\mu\nu}$ that satisfies $\sum_{\nu=0}^{3} g^{\mu\nu} g_{\nu\rho} = \delta^\mu{}_\rho$.

(b) Use the formula

$$\Gamma^\rho_{\mu\nu} = \sum_{\sigma=0}^{3} \frac{1}{2} g^{\rho\sigma} \left[\frac{\partial g_{\sigma\mu}}{\partial x^\nu} + \frac{\partial g_{\sigma\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right] \tag{4}$$

to calculate Γ^0_{ij} and Γ^i_{0j}.