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DUE ON 18.04.2016

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## Homework - 3

1. The current (critical) density of our universe is  $\rho_c \approx 10^{-26} \text{kg/m}^3$ . Assume the universe is filled with cubes with equal size that each contain one person of  $m = 100 \text{kg}$ . What would the length of the side of such a cube have to be in order to give the correct critical density? How many hydrogen atoms would you need in a box of  $1 \text{m}^3$  to reach the critical density? Deep space is very empty and a much better vacuum than we can obtain on earth in a laboratory.

The following two problems are fairly easy, if you are using a computational software like Mathematica. If you do not have access to such a software, you can for example go to <http://www.wolframalpha.com/>. There you can type things like “ $a(t) = a_0$ ”, where  $a(t)$  is an explicit function. The website will then give you the solution  $t = t_0$  that satisfies  $a(t_0) = a_0$ . You can also type “Second derivative of  $a(t)$ ” where  $a(t)$  is again an explicit function. If you are curious you can also “plot  $a(t)$ ” to see how it differs from pure matter  $a(t) \propto t^{\frac{2}{3}}$  and how our universe will eventually expand exponentially.

You can also solve the two problems below without the help of a computer: If you define  $x = c_* e^{\frac{3}{2}\sqrt{c_2}t}$ , then you can rewrite  $a(t) = a_0$  as a quadratic equation in  $x$ . Likewise, after simplifying  $\ddot{a}(t) = 0$  can be rewritten as a quadratic equation.

2. As we learned in the lecture, in our current universe we have  $K/a_0^2 \approx 0$ ,  $\Omega_{\Lambda,0} \approx .692$ ,  $\Omega_{m,0} \approx .308$  and  $\Omega_{r,0} \approx 0$ . In this case one can still solve the Friedmann equation analytically (see below)! Choose your time such that  $a(t=0) = 0$ . Then find using  $H_0 \approx 67.8 \frac{\text{km}}{\text{s Mpc}}$  the correct age of our universe with a three digit precision. (The last digit of this answer can change, if future experiments lead to a slightly different central value for  $H_0$ .)

*Hint:* You can use that the differential equation

$$\dot{a}(t) = \sqrt{\frac{c_1}{a(t)} + c_2 a(t)^2} \quad (1)$$

has the solution

$$a(t) = \frac{e^{-\sqrt{c_2}t} (c_*^2 e^{3\sqrt{c_2}t} - c_1 c_2)^{\frac{2}{3}}}{(2 c_2 c_*)^{\frac{2}{3}}}, \quad (2)$$

where  $c_*$  is the integration constant.

3. Use your result for  $a(t)$  from the above problem and determine *how long ago* the accelerated expansion of our universe started. Now determine the time  $t_{eq}$  at which matter and dark energy contributed equally to the energy density:

$$\Omega_m(t_{eq}) = \Omega_{\Lambda}(t_{eq}) = .5. \quad (3)$$

*How long ago* was that?