Name: DUE ON 25.04.2016

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Homework - 4

1. In lecture 3 we defined the redshift parameter $z \equiv \frac{a(t_0)}{a(t_1)} - 1$. We also discussed that our universe has roughly $\Omega_{\Lambda,0} \approx .7$, $\Omega_{m,0} \approx .3$ and $\Omega_{r,0} \approx 10^{-4}$.
   (a) Use the scaling of the Ω_i with $a(t)$ to determine the redshift parameter at the time of matter-Λ equality: $\Omega_\Lambda = \Omega_m$. As you have seen in the previous homework this was roughly 3.6×10^9 years ago.
   (b) Use the scaling of the Ω_i with $a(t)$ to determine the redshift parameter at the time of matter-radiation equality: $\Omega_m = \Omega_r$. This was roughly 7.5×10^4 years after the big bang.
2. Particle and event horizon in a curvature dominated universe:
   (a) Calculate the particle horizon today at t_0 for a curvature dominated universe with $K = -1$ and $a(t) = a_0 \frac{t}{t_0}$.
   (b) Calculate the event horizon for a curvature dominated universe with $K = -1$ and $a(t) = a_0 \frac{t}{t_0}$.
3. Let us get a rough estimate for the time at which the CMB was released:
   (a) The Boltzmann constant is $k_B = 8.6 \times 10^{-5} eV/K$. Since the photon spectrum has a tail of photons with higher energy, assume that neutral hydrogen could form, when $k_B T_{CMB} \approx .3eV$ (instead of the familar $13.6eV$ ionization energy). Calculate the value of the scale factor $a(t)$ at this time.
   (b) The hydrogen recombination happens still in the matter dominated era. Assume a matter dominated universe $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ and determine the time of 'last scattering', after which the CMB photons could stream freely.
4. A single component universe:
   (a) Rewrite the scale factor for a matter dominated universe $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ in terms of the conformal time τ.
   (b) Rewrite the scale factor for a radiation dominated universe $a(t) = a_0 \sqrt{\frac{t}{t_0}}$ in terms of the conformal time τ.
   (c) Rewrite the scale factor for a universe that is dominated by a cosmological constant and has $a(t) = a_0 e^{H(t-t_0)}$ in terms of the conformal time τ.