Cosmology and particle physics

Lecture notes

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Lecture 3 Our universe (and its fate)

In this lecture we discuss the observed values for the different forms of energy and matter in our universe. Based on that it is easy to discuss the fate of our universe in the far distant future. It will be however much more interesting and complicated to describe the history of our universe from the beginning until today and that is what is going to occupy us for the rest of this semester.

1 Critical density

In a universe like our own the curvature $|K/a_0^2| \ll (\dot{a}(t_0)/a_0)^2$ is very small and it is useful to define the so called critical energy density. From the Friedmann equation

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3}\rho(t)\,, \tag{1}$$

we find after setting $K = 0$ the critical energy density

$$\rho_c(t) = \frac{3H(t)^2}{8\pi G}\,. \tag{2}$$

This critical density today is roughly $\rho_c(t_0) \approx 10^{-26} kg/m^3$ which is incredibly small.

Having defined the critical density, we can normalize the energy density for all fluids by dividing by the critical density and define

$$\Omega_i(t) \equiv \frac{\rho_i(t)}{\rho_c(t)} \quad \text{with} \quad i = m, r, \Lambda\,. \tag{3}$$

We can also define the total normalized energy density $\Omega_T = \sum_i \Omega_i$. With that we can rewrite equation (1) as

$$\Omega_T(t) = \sum_i \Omega_i(t) = 1 + \frac{K}{\dot{a}(t)^2}\,. \tag{4}$$

From this we see that for an open universe $\Omega_T < 1$ and for a closed universe $\Omega_T > 1$. For a flat universe with $K = 0$ we have $\Omega_T = 1$ being constant. Note, that observations will always have uncertainties. This means that if our universe is actually flat ($K = 0$) then we would never be able to know for sure. However, if our universe would have non zero curvature $K/\dot{a}(t)^2 \neq 0$, then this is something we could in principle measure with very high confidence.

2 Our universe

In the last few decades our understanding of the universe has substantially improved and cosmology has become a precision science where most parameters can be measured with error bars that are a few percent or even less. This 'golden age' of cosmology is far from over and future experiments promise substantially better measurements and hold the prospect of discovering new fascinating features of our universe.

As we have discussed before, $|K/\dot{a}_0^2| \ll 1$ in our current universe and there is no observational evidence for non-vanishing curvature. The current upper bound is

$$\left|\frac{K}{\dot{a}_0^2}\right| < .005\,. \tag{5}$$

As discussed above, this means that the total energy density of our universe is very close to the critical energy density. This then for example means that if a particular type of matter or energy constitutes for example 70% of the energy density in our current universe, then its current energy density is $.7\rho_c(t_0)$.

Let us now take inventory of our current universe:

- **Matter:** While matter like the stars in the galaxies are the most obvious form of matter one can think of, they turn out to be actually only a minimal fraction of the matter in our universe (roughly 0.5%). However, there is a lot of Hydrogen and Helium in the universe in large clouds that contribute to what is usually called *baryonic matter*. The reason for this name is presumably that the baryons (protons and neutrons) make up for almost the entire amount of the mass and the leptons (electrons) only contribute a very small amount of mass. The current value for the baryonic density parameter is

  $$\Omega_{b,0}h^2 = .02226 \pm .00023 \quad \Rightarrow \quad \Omega_{b,0} \approx .048\,. \tag{6}$$

  There is another form of matter that only recently became non-relativistic which are neutrinos. They contribute roughly 0.3% of the total energy density. As we will discuss later, during the early times of the universe neutrinos behaved as radiation and not as matter.

  So the matter we know and understand constitutes only roughly 5% of the total energy density of our current universe!

  It turns out that there has to be another type of non-relativistic matter in our universe in order to explain for example the mass difference between the mass of the visible matter in galaxies and the total mass derived from gravitational effects. Since this matter is not visible because it doesn't interact with photons, it is called *dark matter*. We still don't know what this dark matter really is and have not yet been able to detect dark matter particles in any of the ongoing experiments. Nevertheless, we can conclude from cosmological observations that their contribution to the density parameter is

  $$\Omega_{c,0}h^2 = .1186 \pm .0020 \quad \Rightarrow \quad \Omega_{c,0} \approx .258\,. \tag{7}$$

  So the energy density of our current universe arises to roughly 30% from non-relativistic matter with an equation of state parameter $w = 0$. The exact current bound is

  $$\Omega_{m,0} = .308 \pm .012\,. \tag{8}$$

- **Radiation:** We know that there are photons (light) in our universe, however, these contribute a negligible amount to the current energy density. In particular, the photons from the cosmic microwave background, that will play a very important role in the coming lectures, contribute to the density parameter only

$$\Omega_{r,0} \approx 10^{-4} . \tag{9}$$

  Our universe also contains gravitons (gravitational waves) that contribute to the radiation. These gravitons have not yet been detected but several experiments are looking for them. Their contribution to the radiation density parameter today is also negligible.

  So we find that radiation is unimportant in our current universe. However, due to its equation of state parameter $w = 1/3$ we derived $\Omega_r(t) \propto a(t)^{-4}$. This means that in the early universe where $a(t)$ was much smaller, radiation will actually be the dominating form of energy.

- **Dark Energy:** The largest contribution to the density parameter in our current universe is due to *dark energy*, a currently not fully understood form of energy with negative pressure that leads to an accelerated expansion of our universe. We will discuss this in more detail in the next section. Here let us just say that dark energy is very compatible with a cosmological constant Λ with equation of state $w = -1$. The current contribution to the density parameter is

$$\Omega_{\Lambda,0} = .692 \pm .012 . \tag{10}$$

So we have seen that our current universe has negligible curvature and negligible contributions from radiation. The total energy density splits into roughly 70% dark energy (behaving like a cosmological constant) and 30% non-relativistic matter. The fact that most of this non-relativistic matter is made out of unknown particles might come as a big surprise. It often leads to the statement that we don't understand 95% of our universe since we don't understand the dark energy either, however, we know the equations of state parameters for dark matter and dark energy very well, so that we can describe the history of our universe very accurately. As we will see next time, compared to radiation both dark matter and dark energy become less and less important at earlier times so that they actually are unimportant in our description of the very early universe (the same is true for curvature, i.e. the extra contribution in the first Friedmann eqn. for non-zero K).

The different contributions to our universe are summarized in figure 1, whose values slightly differ from the ones I have given above. The reason is that contributions from dark energy and matter can still change at the level of a few percent since they are sensitive to the value of the Hubble constant that has not yet been measured that accurately (recall that this is the reason why we defined it in terms of an unknown h as $H_0 = 100h\, km/(sMpc)$). In the last few years experiments have measured central values of h ranging from .74 to .67. If you think that this is not overly precise you have to keep in mind that in particular satellite experiments take a very long time from the planning stage until the data is analyzed and usually different experiments determine cosmological parameters in very different ways. So we should be happy that they all seem so close to each other that they are mutually consistent. Also, as we will see in the next section, the discovery of dark energy, that is the dominating form of energy in the current universe, was less than 20 years ago!

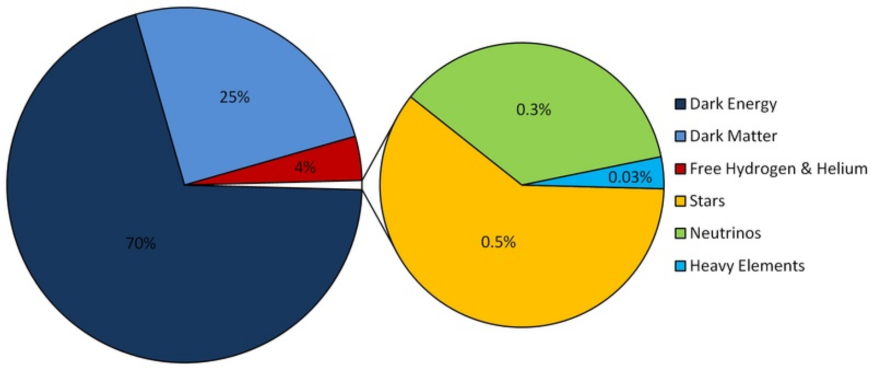

Figure 1: Pie-Chart of the matter and energy content of our universe (taken from Wikipedia).

3 Dark energy

Arguably the most important discovery in cosmology in the last twenty years is the discovery of dark energy. In 1998 the High-Z Supernova Search Team and in 1999 the Supernova Cosmology Project published their analysis of type Ia supernovae, which are a type of standard candles in cosmology. Their observation are in strong tension with a matter dominated universe and much more compatible with a universe whose expansion is accelerating. For this discovery S. Perlmutter, B. Schmidt and A. Riess were awarded the 2011 Nobel Prize in Physics. Before we look at their data, we need to review several useful definitions.

3.1 Redshift

As we have discussed previously, in an expanding universe the wavelength λ of a photon gets stretched as well. For example, if a photon is emitted at time t_1 with wavelength λ_1 and we observe it today at time t_0 with wavelength λ_0, then we have the simple relation

$$\lambda_0 = \frac{a(t_0)}{a(t_1)}\lambda_1\,. \tag{11}$$

Note, in particular for an expanding universe we have $a(t_0) > a(t_1)$ so that the wavelength becomes larger $\lambda_0 > \lambda_1$. This means that the observed light is 'redshifted'. This terminology arises from the fact that for visible light the red wavelengths are the longest. Since bluish light has the shortest wavelengths one likewise uses the term 'blueshift', if wavelength become shorter. This would happen, if the universe contracts or a star is moving towards us with a speed that overcompensates the redshift from the expansion of the universe.

The fractional shift in the wavelength of photons is the so called redshift parameter

$$z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1\,. \tag{12}$$

Since we know the time evolution of $a(t)$ in our universe, we have a one-to-one map from t to $a(t)$ so we can assign to events either a time t or a particular value of the scale factor

$a(t)$. Likewise we can use equation (12) to assign a redshift z to a particular time t_1 in the past. For example, the oldest observed galaxy has a redshift parameter of $z_G \approx 8.6$ which corresponds to a time of only 600 million years after the big bang. So for our stars and galaxies the values of z are rather modest, however, for the cosmic microwave background that will play an important role in the following lectures the redshift is $z_{CMB} \approx 1000$.

As you might have noticed from the definition above in equation (12), the redshift parameter tells us how much smaller the universe was when the light was emitted. We find

$$\frac{a(t_1)}{a(t_0)} = \frac{1}{1+z} \quad \Rightarrow \quad \frac{a(t_1)}{a(t_0)} \approx \frac{1}{z} \quad \text{for} \quad z \gg 1\,. \tag{13}$$

So for example, when the first galaxies were formed around $z_G \approx 8.6$ the universe was roughly one tenth of its current size. When the cosmic microwave background was emitted at $z_{CMB} \approx 1000$ the universe was 1/1000 of its current size.

3.2 Accelerated Expansion

The Supernova Cosmology Project studied 42 type Ia supernova with redshift parameter between $z \approx .2$ and $z \approx .9$. The result is shown in figure 2. [1]

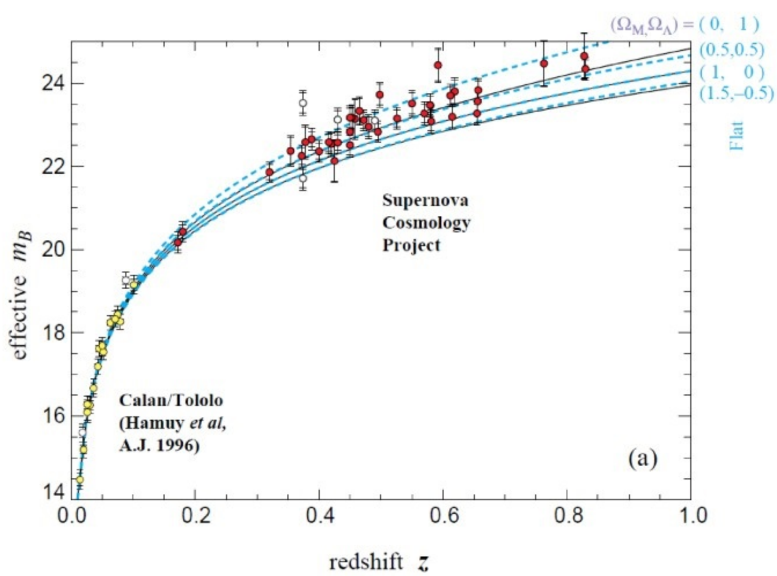

Figure 2: Hubble diagram obtained from 42 high-redshift type Ia supernovae from the Supernova Cosmology Project and 18 low redshift supernovae from the Calan-Tololo Supernova Survey. We see that the observed expansion favors a universe with matter and a cosmological constant.

[1] "effective m_B" denotes the observed brightness measured in a specific wavelength. It is related to the observed flux via $F \propto 10^{-2m_B/5}$ and hence to the distance via $d \propto 10^{m_B/5}$ (see §1.3-1.6 in Weinberg's "*Cosmology*" book for more details).

The data is overlaid with different theoretical curves corresponding to a flat universe with a cosmological constant and non-relativistic matter. We see that the data favors a universe with a substantial contribution from a cosmological constant. When these supernovae were originally studied it was clear that there was severe tension with a flat matter dominated universe and that any kind of energy density that leads to an accelerated expansion would help to explain the discrepancy. While a cosmological constant is the most natural energy density that does the job, it is not the only possibility. From the second Friedmann equation and using $p(t) = w\rho(t)$ we find that any energy with $w < -1/3$ leads to an accelerated expansion:

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}(\rho(t) + 3p(t)) = -\frac{4\pi G}{3}\rho(t)(1+3w) > 0 \qquad \Leftrightarrow \qquad w < -\frac{1}{3}\,. \tag{14}$$

So the equation of state parameter for the dark energy initially didn't have to be $w = -1$ but could have been been very different. However, during the last decade experiments have measured the equation of state parameter for the dark energy very accurately and the current value is

$$w = -1.006 \pm 0.045\,. \tag{15}$$

This is very much consistent with a cosmological constant.

Assuming that the dark energy has equation of state parameter $w = -1$ one can fit the theoretical predictions of a universe with matter and cosmological constant with the data from supernovae (SNe), the cosmic microwave background (CMB) and baryon acoustic oscillations (BAO) to determine the density parameters as shown in figure 3.

3.3 The smallness of the cosmological constant

Interestingly the value of the cosmological constant differs from its natural value by a factor of 10^{-120}, which is certainly the biggest discrepancy between theoretical expectation and measured value that we have ever observed in nature. As you checked in the first homework, if the cosmological constant would be anywhere near its natural value no structures could have formed and such a universe would be empty and lifeless. It actually turns out that the value of the cosmological constant can't be that much bigger than what we observe, since otherwise structure formation would not take place. Concretely, the cosmological constant can only be larger by a factor of roughly 200 since otherwise Hydrogen and Helium clouds would not have clumped to form stars and galaxies.

This factor of 200 does not sound too bad compared to 10^{-120}, however, this fact by itself does not explain the smallness of the observed value. For such an explanation we would first have to assume that there are a gigantic number of universes with random values for the cosmological constant and if this were the case then we would of course live in one that allows for lifeforms to exist. If the cosmological constant is somewhat randomly distributed between zero and the Planck scale then we would need of the order of 10^{120} different universes for this explanation to work! While this sounds crazy, it seems actually very likely that our best candidate for a theory of quantum gravity, which is called string theory, does indeed have so many (or actually many more) solutions.

This *anthropic argument* is pretty unsatisfying. Essentially we didn't explain or derive the smallness of the cosmological constant at all from any underlying theory. However, it

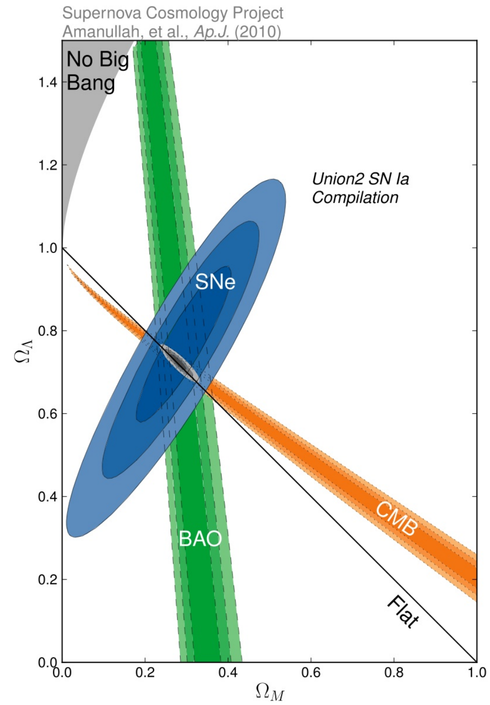

Figure 3: Different experiments exclude different parameter regions leaving only a very small part in the $(\Omega_m, \Omega_\Lambda)$-plane. These three observation are very different and it is very satisfying that all three slices intersect so nicely.

might be that this is how things are. A similar problem is the distance between the earth and the sun. If the earth would be much closer or further away, then there would be no liquid water and life as we know it wouldn't exist. Kepler tried to derive the distance between planets and the sun from an underlying theory. Now we know that Newton's theory of gravity or Einstein's general relativity do not constrain the orbits of the planets but these are rather randomly distributed. Since there are a lot of planets in our solar system and we have also discovered plenty of planets orbiting other stars, the *anthropic argument* in this case is rather common sense. A big difference here is that we can observe other planets. If we would not be able to observe any evidence for the existence of other universes, then this theoretical idea couldn't really be verified.

3.4 The fate of our universe

Now that we know that the current evolution of our universe is governed by a very small cosmological constant let us ask what this means for the future of our universe. As we derived in previous lectures we have $\Omega_m \propto a(t)^{-3}$ and $\Omega_\Lambda = const$. Since $a(t)$ is growing, the matter contribution will become more and more unimportant and our universe is currently entering a phase of exponential expansion. As you can see from equation (5) in lecture 2,

$$\rho(t) \propto a(t)^{-3(1+w)}\,, \tag{16}$$

any contribution to the energy density with $w > -1$, will become less and less important in an expanding universe. We don't have any reliable theoretical models that lead to an equation of state parameter $w < -1$, so it seems very plausible that our universe keeps exponentially expanding in the future. What does that mean?

The first cosmological implication is that the universe lives infinitely long and alternatives like a big crunch are excluded. Since the cosmological constant is so tiny, its implications are otherwise rather minuscule. Concretely, within one year the distance between two objects increases due to the exponential expansion roughly by a modest 0.00000001%. This is so small that the initial gas clouds of hydrogen and helium could clump and form stars, galaxies and galaxy clusters. The 'smaller' structures like our solar system or our galaxy are gravitational bound and will not really experience a different evolution due to the accelerated expansion of our universe. Also the galaxy cluster that contains the milky way will stay gravitationally bound. However, other galaxy clusters that are far away from ours will be redshifted more and more and will eventually become unobservable. This is a somewhat counterintuitive fact that we will make precise next time. Naively one would have expected that we can see more and more of our universe the longer we wait, since light has more time to reach us. But this intuition is wrong in an exponentially expanding universe and in the future we will actually see less and less of our universe!

Having discussed the cosmological fate of our universe based on the current observation, you might be curious about more details and phenomena on smaller scales. For example, stars will eventually burn up all hydrogen and helium and our universe will become a dark place. This doesn't conclude the evolution of our universe and if you are interested you can for example consult Wikipedia for the details of the "Future of an expanding universe".

There is a caveat to the above described fate of our universe. As we will see later, in order to describe inflation we will introduce a scalar field that moves in a potential. The cosmological constant can be explained by such a scalar field that sits at a minimum of the potential, where the value of the potential at the minimum is the value of the cosmological constant. In this alternative description, we can then ask whether this minimum is a local or a global minimum. If it is a local minimum, then the scalar field could tunnel quantum mechanically to another minimum with a smaller cosmological constant. This cosmological constant could be zero or even negative. Such transitions are highly suppressed but if they would happen in the future, then this would of course change the evolution of the universe.