Name: DUE ON 04.04.2017

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Homework - 3

1. The current (critical) density of our universe is $\rho_c \approx 10^{-26} kg/m^3$. Assume the universe is filled with cubes with equal size that each contain one person of $m = 100kg$. What would the length of the side of such a cube have to be in order to give the correct critical density? How many hydrogen atoms would you need in a box of $1\,m^3$ to reach the critical density? The matter we know, which consists mostly of hydrogen, constitutes only 4.8% of the current critical energy density of our universe. So how many hydrogen atoms are actually in a box of $1\,m^3$ in our universe? Deep space is very empty and a much better vacuum than we can obtain on earth in a laboratory.

2. Dark energy constitutes 69% of the current critical energy density in our universe. Calculate the total energy coming from the dark energy inside a sphere around the sun with radius $R_{earth} = 1au$. Compare this to the energy density of the sun $E = M_{sun}$. You see that dark energy does not play a significant role in our solar system.

The following two problems are fairly easy, if you are using a computational software like Mathematica. If you do not have access to such a software, you can for example go to `http://www.wolframalpha.com/`. There you can type things like "$a(t) = a_0$", where $a(t)$ is an explicit function. The website will then give you the solution $t = t_0$ that satisfies $a(t_0) = a_0$. You can also type "Second derivative of $a(t)$" where $a(t)$ is again an explicit function. If you are curious you can also "plot a(t)" to see how it differs from pure matter $a(t) \propto t^{\frac{2}{3}}$ and how our universe will eventually expand exponentially.
You can also solve the two problems below without the help of a computer: If you define $x = c_* e^{\frac{3}{2}\sqrt{c_2}\,t}$, then you can rewrite $a(t) = a_0$ as a quadratic equation in x. Likewise, after simplifying $\ddot{a}(t) = 0$ can be rewritten as a quadratic equation.

3. As we learned in the lecture, in our current universe we have $K/a_0^2 \approx 0$, $\Omega_{\Lambda,0} \approx .692$, $\Omega_{m,0} \approx .308$ and $\Omega_{r,0} \approx 0$. In this case one can still solve the Friedmann equation analytically (see below)! Choose your time such that $a(t = 0) = 0$. Then find using $H_0 \approx 67.8 \frac{km}{s\,Mpc}$ the correct age of our universe with a three digit precision. (The last digit of this answer can change, if future experiments lead to a slightly different central value for H_0.)

   Hint: You can use that the differential equation

   $$\dot{a}(t) = \sqrt{\frac{c_1}{a(t)} + c_2\, a(t)^2} \tag{1}$$

   has the solution

   $$a(t) = \frac{e^{-\sqrt{c_2}\,t}\left(c_*^2 e^{3\sqrt{c_2}\,t} - c_1 c_2\right)^{\frac{2}{3}}}{(2\,c_2 c_*)^{\frac{2}{3}}}\,, \tag{2}$$

   where c_* is the integration constant.

CONTINUED ON THE OTHER SIDE

4. Use your result for $a(t)$ from the above problem and determine *how long ago* the accelerated expansion of our universe started. Now determine the time t_{eq} at which matter and dark energy contributed equally to the energy density:

$$\Omega_m(t_{eq}) = \Omega_\Lambda(t_{eq}) = .5\,. \tag{3}$$

How long ago was that?