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DUE ON 25.04.2017

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## Homework - 4

1. In lecture 3 we defined the redshift parameter  $z \equiv \frac{a(t_0)}{a(t_1)} - 1$ . We also discussed that our universe has roughly  $\Omega_{\Lambda,0} \approx .7$ ,  $\Omega_{m,0} \approx .3$  and  $\Omega_{r,0} \approx 10^{-4}$ .
  - (a) Use the scaling of the  $\Omega_i$  with  $a(t)$  to determine the redshift parameter at the time of matter- $\Lambda$  equality:  $\Omega_{\Lambda} = \Omega_m$ . As you have seen in the previous homework this was roughly  $3.6 \times 10^9$  years ago.
  - (b) Use the scaling of the  $\Omega_i$  with  $a(t)$  to determine the redshift parameter at the time of matter-radiation equality:  $\Omega_m = \Omega_r$ . This was roughly  $7.5 \times 10^4$  years after the big bang.
2. Particle and event horizon in a curvature dominated universe:
  - (a) Calculate the particle horizon today at  $t_0$  for a curvature dominated universe with  $K = -1$  and  $a(t) = a_0 \frac{t}{t_0}$ .
  - (b) Calculate the event horizon for a curvature dominated universe with  $K = -1$  and  $a(t) = a_0 \frac{t}{t_0}$ .
3. The big rip: Observations are compatible with an equation of state parameter  $w < -1$  for the dark energy in our universe. Take  $K = 0$  and a universe filled with matter with density parameter  $\Omega_{m,0}$  and phantom energy  $\Omega_{p,0} = 1 - \Omega_{m,0}$  with  $w_p < -1$ .
  - (a) At what scale factor  $a_{mp}$  are the matter energy density and the phantom energy density equal?
  - (b) Write down the first Friedmann equation for such a universe in the limit  $a(t) \gg a_{mp}$ . Integrate it for an expanding universe and show that  $a(t)$  goes to infinity at a finite cosmic time  $t_{rip}$  given by the relation
$$H_0(t_{rip} - t_0) \approx \frac{2}{3|1 + w_p|\sqrt{1 - \Omega_{m,0}}}. \quad (1)$$
  - (c) The inverse Hubble parameter  $1/H(t)$  controls the size of causally connected patches. What happens to the size of such patches for  $t \rightarrow t_{rip}$ ?
4. Let us get a rough estimate for the time at which the CMB was released:
  - (a) The Boltzmann constant is  $k_B = 8.6 \times 10^{-5} eV/K$ . Since the photon spectrum has a tail of photons with higher energy, assume that neutral hydrogen could form, when  $k_B T_{CMB} \approx .3 eV$  (instead of the familiar  $13.6 eV$  ionization energy). Calculate the value of the scale factor  $a(t)$  at this time.

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- (b) The hydrogen recombination happens still in the matter dominated era. Assume a matter dominated universe  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$  and determine the time of ‘last scattering’, after which the CMB photons could stream freely.

5. A single component universe:

- (a) Rewrite the scale factor for a matter dominated universe  $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$  in terms of the conformal time  $\tau$ .
- (b) Rewrite the scale factor for a radiation dominated universe  $a(t) = a_0 \sqrt{\frac{t}{t_0}}$  in terms of the conformal time  $\tau$ .
- (c) Rewrite the scale factor for a universe that is dominated by a cosmological constant and has  $a(t) = a_0 e^{H(t-t_0)}$  in terms of the conformal time  $\tau$ .