Name: DUE ON 23.05.2017

Matrikelnummer: **NO CLASS ON 30.05.2017**

# Homework - 8

1. (a) Show explicitly that the Boltzmann equation as given in equation (5) in the lecture 8 notes can be rewritten as given in equation (6).

   (b) Assume that $n_i = n_i^{eq}$ for $i = 2, 3, 4$. According to equation (5) how does $n_1 a^3$ evolve in the three cases $n_1 < n_1^{eq}$, $n_1 > n_1^{eq}$ and $n_1 = n_1^{eq}$?

2. Consider the following process in our early universe at $T \gg m_f$:

$$f + \bar{f} \leftrightarrow \gamma + \gamma \,, \tag{1}$$

   where $f$ denotes a fermion and $\bar{f}$ the corresponding anti-fermion.

   (a) If the fermion chemical potential is given by $\mu_f$, what is the anti-fermion chemical potential $\mu_{\bar{f}}$ in thermal and chemical equilibrium?

   (b) Assume that the fermions are quarks in the early universe. After baryogenesis the number of quarks minus anti-quarks is non-zero and does not change. Calculate the difference between the quark and anti-quark number densities for non-zero chemical potential

$$n_q - n_{\bar{q}} = \frac{g}{2\pi^2} \int_0^\infty dp \, p^2 \left( \frac{1}{e^{\frac{p-\mu}{T}} + 1} - \frac{1}{e^{\frac{p+\mu}{T}} + 1} \right) . \tag{2}$$

   You see that the exact expression is fairly simple and that it is indeed non-vanishing for non-zero chemical potential $\mu$.

   Since $a^3(n_q - n_{\bar{q}}) = T^{-3}(n_q - n_{\bar{q}})$ is constant, we find that $\mu$ is a function of the temperature, which is why one usually works with ratios of number densities in which the chemical potentials cancel. (The dependence of $\mu$ on $T$ in the non-relativistic case, is more complicated.)

   *Hint:* Don't worry about the value of $g$. You can use that

$$\int_0^\infty dx \frac{x^2}{e^{x-c} + 1} = -2Li_3(-e^c)$$

   and that the polylogarithm satisfies

$$Li_3(-e^{-x}) - Li_3(-e^x) = \frac{x}{6}(\pi^2 + x^2) \,.$$

3. The early universe is radiation dominated.

   (a) Determine the scaling of the time $t$ as a function of the temperature $T$.

   (b) At a temperature of $T = 1MeV$ our universe is approximately 1.4 seconds old. Determine the time in seconds at which the deuterium abundance is comparable to the abundance of protons, i.e. at which $T \approx .066MeV$.