Name: DUE ON 19.3.2018

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Homework - 1

1. Curvature part 1: Suppose you are a two-dimensional being that is confined to live on a 2-sphere of radius R. If you draw a circle of radius r, its circumference is given by $C = 2\pi R \sin(r/R)$.

   (a) Check that this reduces to the usual expression for very small circles $r \ll R$. Calculate the r value that maximizes C.

   (b) Assume the earth is a perfect sphere with $R = 6371km$. If you could measure C with an accuracy of $1mm$, a circle with what radius would you have to draw to discover that 'your two dimensional universe' is curved?

2. Curvature part 2: The sum of the angles in a triangle in a homogeneous and isotropic space is given by $\alpha_1 + \alpha_2 + \alpha_3 = \pi + \frac{KA}{R^2}$. Here A is the area of the triangle and R the curvature radius. Check this formula for a 2-sphere and a triangle with three angles of $\pi/2$ each.

3. A simple toy universe: Let us solve the Friedmann equations in the presence of only a cosmological constant:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a^2} = \frac{\Lambda}{3}\,, \qquad (1)$$

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3}\,. \qquad (2)$$

   (a) Show that in this particularly simple case the second equation is implied by the first.

   (b) Set $K = 0$ and assume that $\Lambda > 0$. Determine $a(t)$ in an expanding universe. On dimensional grounds the cosmological constant should be $\Lambda = \lambda \cdot \frac{c^5}{\hbar G} \approx \lambda \cdot 10^{86}/s^2$. We don't know of any reason why λ should be particularly small so let us set $\lambda = 1$. Assume two atoms are initially separated by a distance of $d_0 = 10^{-10}m$. How long does it take in this expanding universe until their distance equals the distance between the earth and the center of our galaxy? You just discovered the cosmological constant problem. In our universe λ turns out to be incredibly small $\lambda \approx 10^{-120}$.

   (c) Solve the Friedmann equations for $K = -1$ and a negative cosmological constant. Fix the initial condition so that $a(t = 0) = 0$. Sketch $a(t)$, remembering that the scale factor is determining distances and therefore has to satisfy $a(t) \geq 0$. The fate of such a universe is often called a big crunch.