Name: DUE ON 16.04.2017

Matrikelnummer: **NO CLASS ON 09.04.2018**

Homework - 2

1. Einstein's static universe:

   (a) Suppose that some of the matter in Einstein's static universe is converted by stars into radiation. What will happen to the static universe? Explain your answer.

   (b) Assume $\rho_m = 10^{-26} kg/m^3$, which is the critical density of our current universe. Determine a for such a static universe. How long does light need to travel once through such a universe?

2. Assume our universe (with $K = 0$) would be only filled with radiation with equation of state $w = \frac{1}{3}$. What would its age be? What about a universe that is dominated by curvature with $K = -1$ ($\rho = p = 0$)?

Let us derive the Friedmann equations from classical mechanics and thermodynamics:

3. For a test particle of mass m at a radius $a(t)$ from the origin, Newton's second law yields

$$F = m\ddot{a}(t) = -\frac{GMm}{a(t)^2}\,. \tag{1}$$

   Here M is the total mass of all matter contained inside the sphere of radius $a(t)$. Multiply both side by $\dot{a}(t)$ and integrate. Calculate M, assuming a uniform mass distribution ρ inside the sphere. Choose the constant of integration such that you find the first Friedmann equation

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3}\rho\,. \tag{2}$$

4. Derive the continuity equation from the first law of thermodynamics

$$dE = -pdV + TdS\,. \tag{3}$$

   If the particle interactions in a fluid are small, then we may neglect the heat transfer and set $TdS = 0$. Look at a volume $V = a(t)^3$ and derive the continuity equation.

5. Use your results from 3. and 4. to derive the second Friedmann equation

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}(\rho(t) + 3p(t)\,. \tag{4}$$