Name: DUE ON 30.04.2018

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Homework - 4

1. In lecture 3 we defined the redshift parameter $z \equiv \frac{a(t_0)}{a(t_1)} - 1$. We also discussed that our universe has roughly $\Omega_{\Lambda,0} \approx .7$, $\Omega_{m,0} \approx .3$ and $\Omega_{r,0} \approx 10^{-4}$.

   (a) Use the scaling of the Ω_i with $a(t)$ to determine the redshift parameter at the time of matter-Λ equality: $\Omega_\Lambda = \Omega_m$. As you have seen in the previous homework this was roughly 3.6×10^9 years ago.

   (b) Use the scaling of the Ω_i with $a(t)$ to determine the redshift parameter at the time of matter-radiation equality: $\Omega_m = \Omega_r$. This was roughly 7.5×10^4 years after the big bang.

2. Particle and event horizon in a curvature dominated universe:

   (a) Calculate the particle horizon today at t_0 for a curvature dominated universe with $K = -1$ and $a(t) = a_0 \frac{t}{t_0}$.

   (b) Calculate the event horizon for a curvature dominated universe with $K = -1$ and $a(t) = a_0 \frac{t}{t_0}$.

3. The big rip: Observations are compatible with an equation of state parameter $w < -1$ for the dark energy in our universe. Take $K = 0$ and a universe filled with matter with density parameter $\Omega_{m,0}$ and phantom energy $\Omega_{p,0} = 1 - \Omega_{m,0}$ with $w_p < -1$.

   (a) At what scale factor a_{mp} are the matter energy density and the phantom energy density equal?

   (b) Write down the first Friedmann equation for such a universe in the limit $a(t) \gg a_{mp}$. Integrate it for an expanding universe and show that $a(t)$ goes to infinity at a finite cosmic time t_{rip} given by the relation

$$H_0(t_{rip} - t_0) \approx \frac{2}{3|1 + w_p|\sqrt{1 - \Omega_{m,0}}} \,. \qquad (1)$$

   (c) The inverse Hubble parameter $1/H(t)$ controls the size of causally connected patches. What happens to the size of such patches for $t \to t_{rip}$?

4. Let us get a rough estimate for the time at which the CMB was released:

   (a) The Boltzmann constant is $k_B = 8.6 \times 10^{-5} eV/K$. Since the photon spectrum has a tail of photons with higher energy, assume that neutral hydrogen could form, when $k_B T_{CMB} \approx .3eV$ (instead of the familar $13.6eV$ ionization energy). Calculate the value of the scale factor $a(t)$ at this time.

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(b) The hydrogen recombination happens still in the matter dominated era. Assume a matter dominated universe $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ and determine the time of 'last scattering', after which the CMB photons could stream freely.

5. A single component universe:

   (a) Rewrite the scale factor for a matter dominated universe $a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ in terms of the conformal time τ.

   (b) Rewrite the scale factor for a radiation dominated universe $a(t) = a_0\sqrt{\frac{t}{t_0}}$ in terms of the conformal time τ.

   (c) Rewrite the scale factor for a universe that is dominated by a cosmological constant and has $a(t) = a_0 e^{H(t-t_0)}$ in terms of the conformal time τ.