Homework - 6

1. (a) In homework 2, problem 1, you have shown that the current (critical) density of our universe corresponds to 6 Hydrogen atoms per $1m^3$, which is essentially the same as six protons (baryons) per $1m^3$. However, only 5% of the energy density of our current universe is in the form of baryons. From the ratio

$$\frac{n_b}{n_\gamma} = .73 \times 10^{-9}, \tag{1}$$

   determine the number of photons per $1cm^3$ in our current universe. Note that the answer should be in $centimeter^3$.

   (b) From the formula for the number density for photons, as given in equation (11) in the lecture 6 notes, determine the number of CMB photons per $centimeter^3$ today.

   (c) Using equation (14) in the lecture notes 6, calculate the contribution of these huge number of photons to the normalized energy density today $\Omega_{CMB,0}$.

2. (a) Calculate explicitly the contribution of the standard model particles to the number of relativistic degrees of freedoms $g_\star(T \gtrsim 150MeV) = 61.75$ right before the QCD phase transition.

   (b) After the QCD phase transition the only relativistic particles are the electron e, the μ, the three neutrinos, the photon γ and three pions π^0, π^+, π^-. This leads to $g_\star(T \lesssim 150MeV) = 17.25$. All three pions contribute the same number of relativistic degrees of freedom. What is g for one of the pions?

3. As we will discuss next time, the neutrinos and photons decouple in the early universe so that today the neutrinos have a temperature that is different from the temperature of the photons, i.e. the cosmic microwave background. All three neutrinos have however the same temperature. From equation (24)-(26) in the lecture notes 6 and $g_\star(T_{CMB,0}) = 3.36$, calculate the temperature of the neutrinos today (recall that $T_{CMB,0} = 2.725K$ and only photons and the three neutrinos contribute).