Cosmology and particle physics

Lecture notes

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Lecture 5 The thermal universe - part I

In the last lecture we have shown that our very early universe was in a very hot and dense state. During the expansion of the universe this hot 'soup' cooled and underwent a variety of interesting transitions. In the next few lectures we will discuss the thermodynamical evolution of our universe from a split second after the big bang until the release of the cosmic microwave background 380,000 years after the big bang.

We will from now on set the Boltzmann constant equal to one $k_B = 1$ and measure temperatures in eV.

1 The Standard Model of Particle Physics

Currently the world's largest particle accelerator, the Large Hadron Collider, does experiments at an energy of up to 13 TeV. So we understand particle physics up to this energy scale very well. The particles with masses below this scale and their interactions are described by the so called standard model of particle physics, which is a particular quantum field theory. Here we will recall some features of the standard model of particle physics and summarize the relevant terminology before we discuss particle physics in the early universe. Compared to the periodic table that you are probably familiar with from your chemistry class, the content of the standard model of particle physics is fairly simple. We know in total of 13 different particles that constitute all the matter in the standard model of particle physics. These 13 particles interact via three different forces: The electro-magnetic force, the weak force and the strong force, which we will all discuss below.

1.1 The (known) particles in our universe

The 13 particles come in three different groups:

1. The six *leptons* are probably the particles you are most familiar with. They are fermions and have spin $\frac{1}{2}$. They consist of the electron and its cousins the μ- and τ-particle that all carry one unit of negative electric charge. Additionally there are three neutrinos that are called ν_e, ν_μ and ν_τ and that all are electrically neutral. All these six leptons are uncharged under the strong force but they do interact via the weak force.

2. There are six more fermionic spin $\frac{1}{2}$ particles that are called *quarks*. These quarks combine to form the probably more familiar protons and neutrons as well as other particles that we will discuss below. The quarks also come in two groups of three particles: the up, charm and top quarks carry $+\frac{2}{3}$ units of electrical charge and the

down, strange and bottom quarks carry $-\frac{1}{3}$ unit of electric charge. All quarks are charged under the strong force and the weak force.

3. Lastly there is one more particle in the standard model that was predicted a long time ago but only recently discovered in 2012, the Higgs boson which has spin-0. The Higgs particle plays a special role among all particles. It is responsible for giving a mass to the other particles. This process of giving a mass involves a phase transition in our very early universe. This phase transition changes the cosmological constant we discussed last time by a term that is of the order $\lambda_{Higgs} \approx -10^{-65}$. This means that the value of the cosmological constant before this transition λ_{before} needs to cancel with λ_{Higgs} precisely to 55 digits so that $\lambda_{today} \approx 10^{-120} \approx \lambda_{before} + \lambda_{Higgs}$.

All these particles and their masses are shown in figure 1.

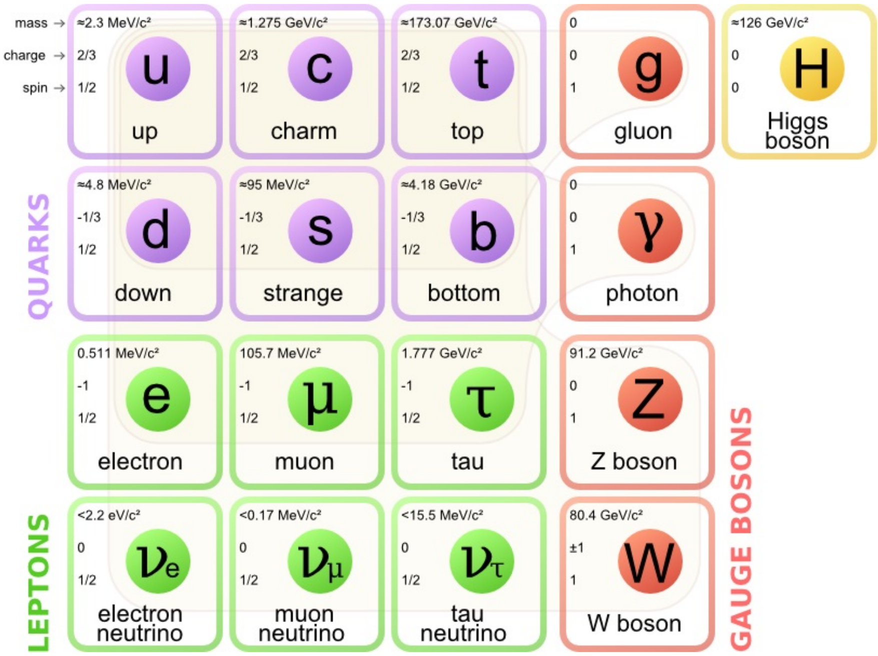

Figure 1: The known particles in our universe (taken from Wikipedia).

Interestingly the leptons and quarks come both in three families (the first three columns in figure 1). Each family contains four particles with the same charges but with different masses. Since the heavier particles in the second and third family can decay into the lighter particles in the first family, it turns out that essentially all standard model particles in our universe are the particles from the first column. [1] This means in particular that all the elements in the periodic table are made up of only of three different particles: the electron, and the up and down quarks.

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[1] Neutrinos can oscillate between different families and heavier particles can be created in processes that involve energies larger than their rest mass but these heavier particles quickly decay to up and down quarks and/or electrons.

1.2 The three forces in the standard model of particle physics

There are four particles that control the interactions between these 13 particles. The three different interactions in the standard model of particle physics are mediated by bosonic spin 1 particles and we quickly discuss the three interactions:

The electro-magnetic force

The force you are probably most familiar with is the electro-magnetic force that is mediated by photons. Many of the particles in the standard model carry an electric charge and therefore interact with photons. Since the photon is massless the interaction strength between two particles falls off as a power law (versus an exponential). This means that the electro-magnetic force could compete with gravity which has the same power law fall off and is in some sense much weaker. However, our universe is essentially electrically neutral on all but very small scales. So for example the earth and the sun carry essentially no net electric charge so their interaction is only determined by their masses, i.e. by gravity, and the electro-magnetic force plays no role. The same is true on larger scales that are relevant for cosmology.

The weak force

The weak interaction is probably the force you are most unfamiliar with. It is mediated by three different particles: the electrically neutral Z boson and the two electrically charged $W^{\pm}$ bosons, where the later carry ±1 unit of electric charge. Since these three particles that mediate the weak force are rather heavy, the weak force is relevant only on fairly short distances, smaller than $10^{-16}m$, like inside a nucleus. There the weak force is responsible for radioactive decay of nuclei.

The strong force

The strong force is what keeps for example the positively charged protons together inside the nuclei. So it is clear that it is much stronger than the electric force between these charged particles. At low energies the strong force becomes so strong that charged particles cannot exist in isolation. Whenever we try to separate two particles that are charged under the strong force, the energy in the field between the particles becomes so large that it can lead to pair production of new particles that combine with the particles we tried to separate into particles that are neutral under the strong force. The strong force is different from the electromagnetic force in the sense that there are three different types of charges (plus their anti-charges carried by the anti-particles). So we cannot label the charges by a positive or negative number but rather we have to use three different labels (and anti-labels). For lack of a better label people call the three charges red, green and blue (and the corresponding anti-charges anti-red, anti-green and anti-blue). All quarks carry a single color charge, i.e. they come in three types so that for example we have a red, a green and a blue up quark. To form particles that are neutral under the strong force we need all three colors to appear once or we can combine a color with an anti-color.

1.3 Hadrons, baryons and mesons

Since the quarks are not playing much of a role in our everyday life, let us discuss a little bit more how they form more familiar composite particles, which also allows us to introduce a little bit more terminology. Above we discussed the six leptons, the electron, muon and tau and the three corresponding neutrinos. The name lepton is derived from a Greek word that means fine, thin, little, which is appropriate since, as far as we know, these particles are fundamental in the sense that they are not composed of other particles. The leptons are supposed to be contrasted with the *hadrons*, derived from the Greek word for thick and strong. These are not fundamental particles but rather particles that are kept together by the strong force. These hadrons are furthermore divided into *mesons*, which are bosonic particles with integer spin and *baryons*, which are fermionic particles with half integer spin.

Let us try to build some baryons. As we have heard above, the quarks have to appear in color neutral bound states at low energies. So we cannot get a baryon that is made up of a single quarks. In order to half integer spin, i.e. a baryon, we therefore need three quarks (and no anti-quarks). If we take three quarks of the same type, like for example three up quarks, then they cannot combine due to the fermi statistics. Since the quarks are spin $\frac{1}{2}$ particles we can combine at most one spin $+\frac{1}{2}$ quark and one spin $-\frac{1}{2}$ quark of the same type. So the lightest baryon is a composite particle of two up quarks and one down quark, often denoted as uud. This baryon has electric charge +1 and is called the proton. One might think that each of the three quarks can have different color charges so there should be more than one proton, however, this is not the case since color neutrality requires us to take the antisymmetric combination of all color neutral combinations of the three quarks. The proton is as far as we know the only stable baryon. The next heavier baryon is udd. It is the electrically neutral neutron, that has a mean lifetime of slightly less than 15 minutes. These two baryons will play a very important role in the creation of nuclei in our very early universe, as we will discuss in lecture 8. All other baryons have a very short lifetime so that they quickly decay into protons and neutrons.

The mesons are all unstable so they do not play such an important role in cosmology. Let us nevertheless discuss the pions that are the lightest mesons: We want to get a particle with integer spin so we need (at least) two quarks to construct a meson. Since we want these particles to be color neutral we need actually a quark and an anti-quark to construct the simplest and lightest mesons. Restricting again to hadrons formed out of the u and d quarks and anti-quarks we seem to have four possibilities to construct two quark mesons: $u\bar{u}$, $d\bar{d}$, $u\bar{d}$ and $\bar{u}d$, where a bar over a quark denotes the anti-quark. However, the actual meson particles we observe are sometimes linear combinations of quark-anti-quark pairs. In particular the sum of $u\bar{u}$ and $d\bar{d}$ combines with $\pm s\bar{s}$ to form two η mesons. This leaves us with only three light π mesons that are made up of u and d quarks and anti-quarks: The $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, the $\pi^+ = u\bar{d}$ and the $\pi^- = \bar{u}d$, where the superscript denotes the electric charges in units of the electron charge. All these mesons decay very quickly with a mean lifetime of $2.6 \times 10^{-8}s$ for the $\pi^\pm$ and $8.4 \times 10^{-17}s$ for the π^0.

1.4 Gravity and a theory of everything

Note, that the standard model or particle physics neglects gravity entirely. This is very well justified in most regimes of interest to elementary particle physics but it tells us that in order to describe our entire universe, we need another theory that unifies quantum field theories with gravity in a so called theory of everything.

General relativity that we are using in this course to describe the evolution of our universe is likewise incomplete since it is a classical theory and it inevitably breaks down near the Planck scale M_P which is given by

$$M_P = \frac{1}{\sqrt{8\pi G}} = 2.435 \times 10^{18} GeV\,. \tag{1}$$

Since the universe was at higher and higher temperatures/energies the further back in time we are going, we reach a point at which general relativity cannot be used anymore. This in particular means we cannot use general relativity to understand the actual beginning of our universe, i.e. the big bang. It is also unclear whether we will ever be able to get experimental insight into the physics that caused the big bang. However, between the energies of less than an eV at the time the CMB was released until the energies studied in particle accelerators of a few TeV we have 12 orders of magnitude of well understood physics to discuss and from the TeV range up to the Planck scale we will discuss another 15 orders of magnitude in energy of slightly more speculative physics. There are also ideas for theories of quantum gravity that go beyond general relativity and might allow us to theoretically understand the initial singularity that arises in general relativity at the beginning of our universe.

2 The universe in thermal equilibrium

In the early universe at temperatures above a few hundred GeV all standard model particles will have energies that are much larger than their rest mass:

$$E(p) = \sqrt{m^2 + p^2} \approx p\,. \tag{2}$$

This means that they do not behave like non-relativistic (pressureless) matter but rather like radiation (i.e. like for example photons for which $E = p$). Since the masses are negligible in this era, there is only one scale in the standard model which is the rate of interactions Γ, i.e. the number of interactions per time. In principle this rate can be different for the different particles but we neglect this for the rough estimates in this section. In our expanding universe there is one more length or time scale set by the Hubble scale H. If the particles interact a lot without feeling the expansion of the universe, then they will be in local equilibrium. This would mean that

$$\Gamma \gg H\,. \tag{3}$$

If the above equation is true, then we can use equilibrium thermodynamics to describe our universe. We would therefore like to estimate during which temperatures/energies the above is expected to be true.

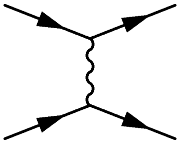
The particle interaction rate can be written as

$$\Gamma \equiv n\sigma v\,, \tag{4}$$

where n is the number density, i.e. the number of particles per volume, σ is the interaction cross-section and v is the average velocity of the particles. Since, as we argued above, all particles are highly relativistic for $T \gg 100 GeV$, we have $v \approx c = 1$. The only dimensionful quantity is the temperature T, that has the dimension of an energy which is the same as an inverse length. So we find for the number density and the cross section

$$n \sim T^3 \,, \qquad \sigma \sim T^{-2} \,. \tag{5}$$

For the cross section we can be more precise. Two particles interact dominantly via the exchange of one of the gauge bosons (that are all massless above $100 GeV$). We often write this in terms of Feynman diagrams that use straight lines to indicate fermions and wiggly lines to describe a gauge boson:



The interaction cross section is the square of such a diagram so that it goes like the fourth power of the interaction strength between the fermion and the gauge boson. This interaction strength is usually called $\sqrt{\alpha}$, which gives

$$\sigma \sim \frac{\alpha^2}{T^2} \,. \tag{6}$$

Putting this together we find the following scaling of the interaction rate

$$\Gamma \approx n\sigma \approx \alpha^2 T \,. \tag{7}$$

The actual value of α depends on the particular force with which the particles interact as well as the energy scale. However, at high energies the interaction strengths of all forces seem to become almost the same, which hints at a unification of all force in a so called grand unified theory (GUT). At this GUT scale the energy is approximately $10^{16} GeV$ and the value of α is $\alpha \approx .05$.

As we have seen last time, our early universe was dominated by radiation. [2] This means that

$$H^2 = \frac{\rho}{3M_P^2} \sim \frac{1}{a(t)^4 M_P^2} \sim \frac{T^4}{M_P^2} \qquad \Rightarrow \qquad H \sim \frac{T^2}{M_P} \,. \tag{8}$$

Putting this together we find that

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_P}{T} \sim \frac{10^{16} GeV}{T} \,, \tag{9}$$

which means for roughly $100 GeV \ll T \ll 10^{16} GeV$ we have $\Gamma \gg H$. So our early and hot universe was in a state of local equilibrium and we can describe it using equilibrium thermodynamics.

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[2] This is also plausible from the above discussion that showed that all the standard model particles behaved like radiation instead of matter in the early universe.

Baryogenesis

In the following we will describe the cooling of our universe starting from a 'soup' of matter and photons at a temperature of a few hundred GeV, using our knowledge of particle physics and thermodynamics. However, before we do that let us mention a puzzle: In a very hot universe we can create particle-anti-particle pairs from photons, denoted γ. For example for the electron e^- and the positron e^+ we can have the reversible process

$$e^- + e^+ \leftrightarrow \gamma + \gamma\,. \tag{10}$$

In an expanding universe we know that the photons 'lose' energy due to the redshift. This means that there is a certain point at which the two photons on the right won't have enough energy to create an electron-positron pair. At that moment the above process should only go in one direction

$$e^- + e^+ \rightarrow \gamma + \gamma\,. \tag{11}$$

If the early universe has an equal number of particles and anti-particles then eventually we would expect that all particles and anti-particles annihilate and leave a universe filled with photons. However, in our universe there is an asymmetry between matter and anti-matter, so that our universe ended up with matter and not just radiation. This asymmetry can be quantified by the ratio between the number of baryons (protons and neutrons) and photons in our current universe. Observation tell us that today

$$\frac{n_b}{n_\gamma} \sim 10^{-9}\,, \tag{12}$$

while the same ratio for anti-baryons seems to be essentially zero. There is no mechanism inside the standard model of particle physics that can explain this so called baryogenesis, i.e. the observed matter-anti-matter asymmetry, so we will simply assume that the initial conditions of the universe were such that they lead to the observed baryon to photon ratio.[3]

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[3] There are a variety of theoretical ideas of how such an asymmetry can arise but so far the experiments have not singled out any particular model, so we refrain from discussing baryogenesis in any further detail.