Cosmology and particle physics

Lecture notes

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Lecture 4 Our universe from 3.8×10^5 to 13.8×10^9 years

1 Particle and event horizon

Let us return to the FRW metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right) . \tag{1}$$

We argued that isotropy forbids mixed terms between the time and spacial coordinates. Then by redefining the time coordinate we can choose the coefficient of dt^2 to be unity. However, there is another very convenient time coordinate, that is called conformal time and we will denote it by τ. It is defined such that

$$dt^2 = a(\tau)^2 d\tau^2 . \tag{2}$$

This means that the FRW metric takes the form

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right) . \tag{3}$$

Note, that we chose $K \in \{-1, 0, 1\}$, so that it is clear from (1) that r, θ, ϕ are dimensionless, while t and $a(t)$ have the dimension of length or time (recall that $c = 1$). This then means that τ is also dimensionless. Likewise $\dot{a}(t)$ is dimensionless and

$$a'(\tau) \equiv \frac{da}{d\tau} , \tag{4}$$

has the dimension of length or time.

For example for $K = 0$ the metric (3) is just the flat space Minkowski metric multiplied by an overall factor $a(\tau)$. Such a factor that multiplies the entire metric is called a conformal factor, hence the name conformal time for τ. As we have seen in the first lecture the coordinates r, θ, ϕ do not give us *physical distances* since they neglect the factor $a(t)^2$ in the metric. The coordinates r, θ, ϕ are called *comoving coordinates*. Many observable objects like 'standard candles' have a non-zero velocity in comoving coordinates: $\vec{v}_{comoving} = a(t)\dot{\vec{r}}$ in addition to the velocity due to the Hubble expansion $\vec{v}_{Hubble} = \dot{a}(t)\vec{r}$. For far away objects the Hubble velocity is usually much larger while for close by objects like for example cepheids in our galaxy, the Hubble velocity is negligible.

Light plays a special role in observations but also in determining the causal structure of our universe since no information can travel faster than light. So two places that cannot exchange light in the lifetime of our universe are causally disconnected. Light follows a null-geodesic which means that $ds = 0$. From the way we have written the metric in equation (3), we see that in this case the scale factor $a(\tau)$ does not matter at all and for example for a radially traveling light ray we have

$$ds = 0 \qquad \Rightarrow \qquad d\tau = \frac{dr}{\sqrt{1 - Kr^2}}\,, \tag{5}$$

independent of $a(\tau)$.

1.1 The particle horizon

Similarly to a black hole, where the event horizon indicates the horizon beyond which observers from the outside cannot see, i.e. from beyond which they cannot receive any light, there are two important horizons in cosmology. The first horizon, which is called *particle horizon* defines the maximal distance a photon can have traveled since the beginning of the universe. In an expanding universe we have to be precise by what we mean by this distance: We mean the current distance at time t or τ between the photon and the object that emitted it at the beginning of the universe. Without loss of generality we can look at a photons starting at the origin $r = 0$ and traveling outward. So we have

$$d_H \equiv a(\tau) \int_0^{r_H} \frac{dr}{\sqrt{1 - Kr^2}} = a(\tau) \int_{\tau_i}^{\tau} d\tau' = a(t) \int_{t_i}^{t} \frac{dt'}{a(t')}\,, \tag{6}$$

where we used (5) and (2).

For example for a matter or radiation dominated universe we have $a(t) = a_0(t/t_0)^p$ with $p < 1$ and the beginning of the universe is at $t_i = 0$. This leads to

$$d_H(t_0) = a_0 \int_0^{t_0} \frac{dt'\, t_0^p (t')^{-p}}{a_0} = \frac{t_0}{1-p} < \infty\,. \tag{7}$$

This means that light can have only traveled a finite distance since the beginning of the universe, which is what we would have naively expected. This of course also means that we can only see a finite part of our universe.

For a universe that is exponentially expanding due to a cosmological constant we have $a(t) = a_0 e^{H(t-t_0)}$ and the beginning of the universe, i.e. $a(t_i) = 0$, is at $t_i = -\infty$. This leads to

$$d_H(t_0) = \int_{-\infty}^{t_0} dt' e^{-H(t'-t_0)} = -\frac{1}{H} e^{-H(t'-t_0)}\Big|_{t'=-\infty}^{t_0} = +\infty\,, \tag{8}$$

so that in this case the particle horizon is infinite. This fact will be tremendously important once we discuss inflation. The reason is that the cosmic microwave background, which was created shortly after the big bang, is essentially the same on distances much larger than the particle horizon of a matter or radiation dominated universe. This seems in contradiction with causality and requires us to postulate a phase of exponential expansion at the beginning of the universe, which is called inflation.

This cosmic microwave background is the first light in our universe that we can still observe today. It originated shortly after the big bang so the light has been traveling for

13.8 $Gyrs$. We can now ask how big the visible universe is today by calculating the event horizon

$$d_H(t_0) = a_0 \int_0^{t_0} \frac{dt'}{a(t')} \,. \tag{9}$$

For that purpose it is sufficient to take the matter and the cosmological constant into account which leads to

$$a(t) = a_0 \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{3}} \left[\sinh\left(\frac{3}{2} H_0 \sqrt{\Omega_{\Lambda,0}} t\right)\right]^{\frac{2}{3}} \,. \tag{10}$$

Plugging in the values from the last lecture $\Omega_{\Lambda,0} = .692$ and $\Omega_{m,0} = .308$ leads to

$$d_H(t_0) \approx 3.4 t_0 \approx 47 Glyrs \,. \tag{11}$$

So our visible universe has currently a radius of $47Glyrs$, although it is only 13.8 $Gyrs$ old. This shouldn't be too surprising since we know that our universe has been constantly expanding since the big bang.

1.2 The event horizon

Another important horizon in cosmology is called the *event horizon*. It refers to the maximal distance light emitted today at t_0 can travel. This horizon determines which parts of space we can exchange information with. If the event horizon is finite, then there are parts of the universe which are causally disconnected from us and similar to the black hole, these parts cannot send information to us (contrary to the black hole, we cannot send information to these parts of the universe either).

The definition of the event horizon is

$$d_e \equiv a_0 \int_{t_0}^{\infty} \frac{dt'}{a(t')} \,. \tag{12}$$

Let us again first look at a matter or radiation dominated universe with $a(t) = a_0 (t/t_0)^p$ and $p < 1$. We find

$$d_e = \int_{t_0}^{\infty} \frac{dt'\, t_0^p}{t'^p} = \left.\frac{t_0^p\, t'^{(1-p)}}{1-p}\right|_{t'=t_0}^{\infty} = \infty \,. \tag{13}$$

Again this result is consistent with our naive expectation. In the infinite time until the end of the universe the light can travel an infinite distance, so that in such a universe we could send and receive signals from anywhere in the universe. However, as we discussed last time our universe is currently and in the future dominated by a cosmological constant and $a(t)$ approaches an exponential expansion $a(t) = a_0 e^{H(t-t_0)}$. As we mentioned last time this means that we can *never* see the entire universe

$$d_e = \int_{t_0}^{\infty} dt'\, e^{-H(t'-t_0)} = -\left.\frac{1}{H} e^{-H(t'-t_0)}\right|_{t'=t_0}^{\infty} = \frac{1}{H} = const \,. \tag{14}$$

So we see that even if the light travels infinitely long it can only tell us about places at a finite distance. Intuitively we can understand this since the exponential expansion constantly stretches the space between two objects. If the distant is larger than $1/H$, then in any given amount of time the increase of the distant due to the stretching is larger than the distant light can travel.

Since our universe has currently still a substantial amount of matter, the Hubble parameter is changing until in the far future the energy density is almost completely given by the dark energy and the Hubble parameter becomes constant. This asymptotic Hubble parameter in our universe is actually not that different from our current value $H(t = \infty) \approx 1.2H_0$. So this means that we can currently exchange information with objects that have a distance of less than $1/H_0 \approx 14.4 Glyrs$ from us, while in the far future we can only exchange information with objects that have a distance of less than $1/H(\infty) \approx 12 Glyrs$.

Since during an exponential expansion objects that are not gravitationally bound to our galaxy will move further and further away from us, they will actually leave our event horizon in the future. This means for example that very distant galaxy clusters that we can exchange information with today will at one point in the future leave our event horizon and become unobservable. As mentioned last time, contrary to intuition, we will therefore be able to exchange information with less parts of our universe in the future.

We can also ask how much of the universe we can ever observe in the future. We have seen that the first light (the CMB) has a current distance that is far bigger than the above event horizon. This is due to the fact that our universe in the past wasn't dominated by dark energy and therefore didn't have a finite even horizon. Once we have a finite event horizon, only light within this horizon can reach us. This means that there is light from far distant objects that has entered our event horizon already but hasn't reached us yet. The maximal distance of such objects that we will be able to see in the future can be calculated as sum of the particle and event horizon

$$d_H(t_0) + d_e = a_0 \int_0^\infty \frac{dt'}{a(t')} \approx 4.6 t_0 \approx 64 Glyrs\,, \tag{15}$$

where we used $a(t)$ from equation (10).

2 The cosmic microwave background (CMB)

In 1964 Arno Penzias and Robert Wilson were working on the detection of radio waves that bounced off echo balloon satellites when they discovered a faint background of radiation in the microwave range. Surprisingly this signal seemed to come from everywhere in the sky. Checking their antenna they discovered a family of nesting pigeons that they removed together with what Penzias called "white dielectric material" (aka bird poop). Nevertheless the signal remained. At the same time some astrophysicists were planning to search for such a signal since they had realized that, if the universe had started in a hot dense state, then the subsequent expansion would lead to photons whose wavelength would get red-shifted due to the expansion of the universe in such a way that their wavelength is today in the millimeter or micrometer range. This is exactly what Penzias and Wilson discovered and for this they were awarded the 1978 Nobel Prize.

In the subsequent decades these photons that are called the cosmic microwave background (CMB) since they fill the universe homogeneously and isotropically have been studied by many ground based and satellite experiments. This CMB is a clear evidence for a hot big bang and at the same time the best tool for precision measurements in cosmology. The COBE satellite that was launched in 1989 was the first space-based experiment that measured the CMB. It showed that the CMB follows the best black-body spectrum ever observed in nature (see figure 1).

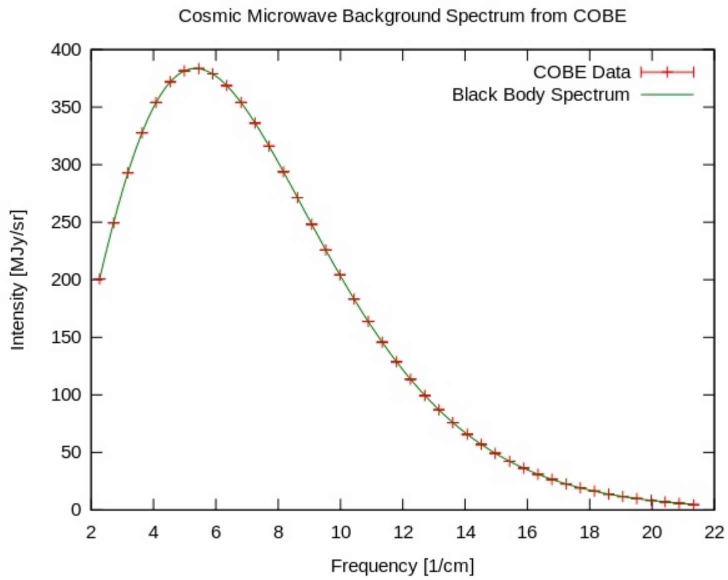

Figure 1: The black body spectrum of the CMB as measured by the COBE satellite. The errors bars are too small to observe and it is impossible to distinguish the theoretical curve from the measured spectrum (figure taken from Wikipedia).

George Smoot and John Mather, two of COBE's principal investigators, were awarded the 2006 Nobel Prize in physics for their work on the COBE project. This shows the great importance of the CMB in understanding the evolution of our universe from the very beginning until today.

Recall that the radiation from a black body is described by Planck's law which gives for $c = \hbar = 1$

$$B(\lambda, T) = \frac{4\pi}{\lambda^5} \frac{1}{e^{\frac{2\pi}{\lambda k_B T}} - 1} \,. \tag{16}$$

Here $k_B = 8.6 \times 10^{-5} eV/K$ is the Boltzmann constant given in terms of elctron volts and Kelvin. The black body spectrum of the CMB corresponds to a temperature of

$$T_{CMB,0} = 2.72548 \pm 0.00057K \approx -270^\circ C \,. \tag{17}$$

So we see that the CMB is pretty cold. However, the wavelength of photons gets red-shifted so that a current wavelength λ_0 was at an earlier time t_1 $\lambda_1 = a(t_1)\lambda_0/a_0$. This tells us that the wavelength scales like $a(t)$ and then it follows from (16) that the temperature of the CMB scales like $1/a(t)$. In particular the temperature of the CMB at the earlier time t_1 is given by

$$T_{CMB}(t_1) = \frac{a_0}{a(t_1)} T_{CMB,0} = (1+z) T_{CMB,0} \,. \tag{18}$$

So we see that the temperature of the CMB was much larger for a much smaller universe. This means that the further we go back in time the hotter the universe was.

We have seen in the last lecture that the CMB radiation is only contributing a very small amount to the current energy density of our universe. Nevertheless, the radiation energy density has the strongest dependence on the scale factor and will therefore inevitably dominate in the very early universe. Recall from the last lecture that the Friedmann equation for our universe can be written as

$$\rho_c(t) = \rho_\Lambda + \rho_m(t) + \rho_{rad}(t) \approx \rho_c(t_0)\left[.7 + .3\left(\frac{a_0}{a(t)}\right)^3 + 10^{-4}\left(\frac{a_0}{a(t)}\right)^4\right]. \tag{19}$$

While the cosmological constant is currently and in the future dominating the energy density, this was different at earlier times when $a(t)$ was much smaller than a_0. In particular, if we plot the energy density as a function of a as a log-log-plot we find the following history of our universe:

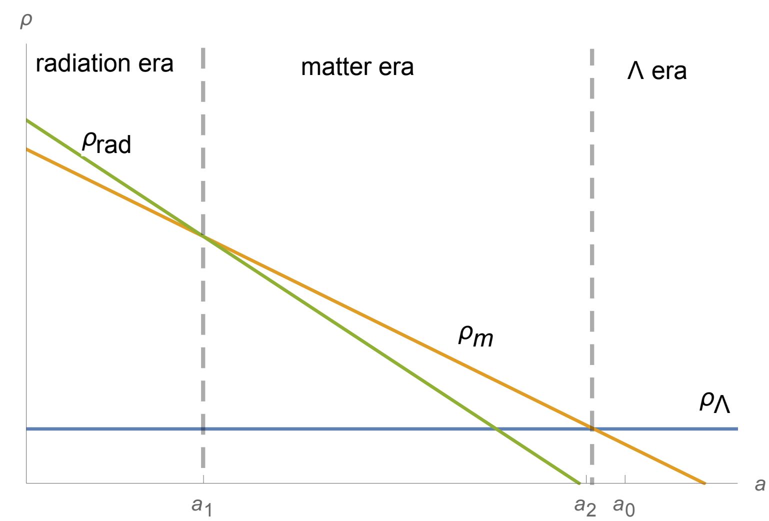

Figure 2: The evolution of our universe.

While we are currently (and in the future) in an era dominated by the dark energy, this was different in the past. The substantial amount of matter in the universe was dominating its evolution until fairly recently. In the far distant past, when the universe was much, much smaller, radiation was actually the dominating form of energy density since it grows like $a(t)^{-4}$.

Recall that a curvature contribution proportional to $K/a(t)^2$ was never the dominating form of energy density. It is currently very small (if not zero) and will become less important in the future since it decays with increasing $a(t)$ while the dark energy is most likely constant. Going backwards in time the matter and radiation contributions will grow faster than the curvature so that the curvature was less important in the past. So the curvature was never dominating but it could nevertheless be non-zero and measurable with more precise experiments in the future. This would then not really affect the part of the evolution of our universe that we are currently discussing but it would be interesting on theoretical grounds and might hint or exclude certain transitions in the very, very early universe.

Above we have argued that the temperature of the CMB is decreasing over time and therefore in the past the universe was much hotter. From equation (19) and the plot 2 we see that the universe in the past also had a much higher energy density. So our universe started out in a much hotter and much denser state that then got diluted and cooled due to the expansion of the universe. This means that we can use thermodynamics and our knowledge of particle and nuclear physics to understand its early evolution.

3 From 380,000 years after the big bang until today

Before we delve into the more involved evolution of the early universe, let us first discuss the evolution from the time the CMB was released until today. The following picture shows the rather few important cosmological periods of our universe since the release of the cosmic microwave background:

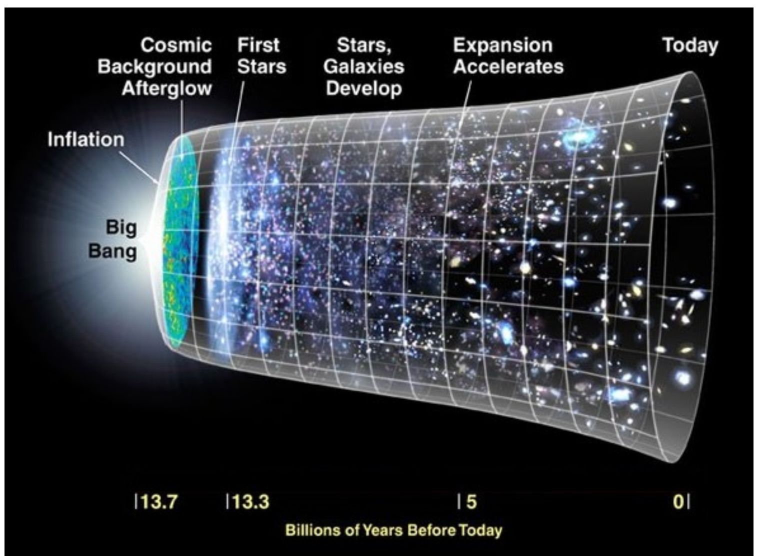

Figure 3: The evolution of our universe since the release of the cosmic microwave background 3.8×10^5 years after the big bang.

As discussed before and studied in the homework, the cosmological constant started to dominate the evolution of the scale factor $a(t)$ a few billion years ago. However, there is substantial amount of baryonic and dark matter in the universe and this form of non-relativistic matter was dominating in the not too distant past.

During this matter dominated era very small deviations from a perfectly homogeneous universe were amplified by gravity and structures started to form. We will later discuss these inhomogeneities in more detail and understand their amazing origin: quantum fluctuations! These very small effects in our every day life have actually led to the small inhomogeneities that are the seeds of our stars and galaxies, which is one of the most amazing features of our universe. As we will discuss in the next lectures a few hundred years after the big bang our universe was a soup of atomic nuclei, electrons and photons. The photons constantly interacted with the electrons via Compton scattering and the negatively charged electrons interacted with the positively charged nuclei via the Coulomb force. Atoms like hydrogen were not stable as long as there were photons with an energy larger than the binding energy of hydrogen which is $13.6eV$, since these photons would ionize the hydrogen. However, at around 380,000 years after the big bang the universe had cooled enough so that stable atoms could form. The photons at this time had mostly energies below the $13.6eV$ threshold and could not ionize the atoms anymore. Since the atoms are electrically neutral their interaction with the photons became negligible at this point and the photons could essentially stream freely. These photons are what we observe today so they have been traveling for more than 1.3×10^{10} years! Detailed studies of the photons and particular the deviation from homogeneity and isotropy tell us a lot about the universe at the time of the so called 'last scattering' after which these photons were able to stream freely. However, these photons also tell us about much earlier times as we will see later in this course.

As mentioned above, when neutral atoms formed roughly $380,000$ years after the big bang, very small inhomogeneities started to amplify due to the gravitational attraction: The initially more or less homogeneous distribution of neutral atoms contained only hydrogen, helium and lithium. Any small inhomogeneity in such a setup will amplify due to gravity: Denser regions will attract more matter and become even denser and therefore increase their gravitational attraction. Such regions that become more and more dense will eventually after a few hundred million years lead to the first stars. In these stars the gravitational attraction is sufficiently strong to start nuclear fusion, which provides the energy for the stars to shine and also leads to the creation of elements heavier than Lithium. These heavier elements were then released at the end of the lifetime of the first stars in supernovae explosions. Stars and planets like the ones in our solar system that are then later on formed contain heavier elements. This star formation process out of the initial hydrogen and helium mixed with some heavier elements will go on for roughly 10^{14} years. This follows since as we have discussed last time a large amount of hydrogen and helium is still in clouds outside of stars.