Cosmology and particle physics

Lecture notes

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Lecture 11 Inflation - part III

Last time we discussed the relevant equations for slow-roll inflation as well as a few concrete slow-roll models. This time we discuss a little bit the experimental status of these models, the reheating process and for the first time in this course the deviation from a completely homogeneous universe.

1 Experimental constrains on inflationary models

As we mentioned last time and as you have seen in the homework, experiments place bounds on existing inflationary models. Several models have already been excluded and future experiments will tighten the bounds and further shrink the parameter space for inflationary models. Ideally this will ultimately single out one particular model of inflation.

There are several ground based experiments that observe a small patch on the sky and have provided lots of important data that constraints the cosmological parameters. Satellites on the other hand have access to most of the sky, however they require that the entire measuring apparatus can be transported into space where it has to work without being maintained or upgraded. Currently the most stringent bounds on inflationary models where released by the Planck collaboration in February 2015 and the BICEP2 and Keck array in October 2015. The Planck satellite has measured the black body spectrum from the CMB at several different frequencies and also the polarization of the photons. Its data tightens most bounds and favors simple single field slow-roll inflationary models. Furthermore, by constraining the slow-roll parameters

$$\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 , \qquad (1)$$

$$\eta_V \equiv M_P^2\frac{V''(\phi)}{V(\phi)} . \qquad (2)$$

it tells us about the form of the potential at a time of 50-60 e-folds before the end of inflation.

The experimental data is usually presented in terms of constrains on the so called spectral index $n_s = 1 - 6\epsilon_V + 2\eta_V$ and the tensor-to-scalar ratio $r = 16\epsilon_V$. While inflationary models generically predict values of $n_s \neq 1$, these values can in principle be larger or smaller than 1. However, the data clearly requires $n_s < 1$. The particular case of $\epsilon_V = 0$, $\eta_V > 0$ which gives rise to $n_s > 1$ is experimentally excluded. This case would have corresponded to a true de Sitter phase, i.e. a positive cosmological constant, instead of a slowly rolling scalar field. The figure below shows the experimental constraints in the (n_s, r)-plane together with a variety of inflationary models:

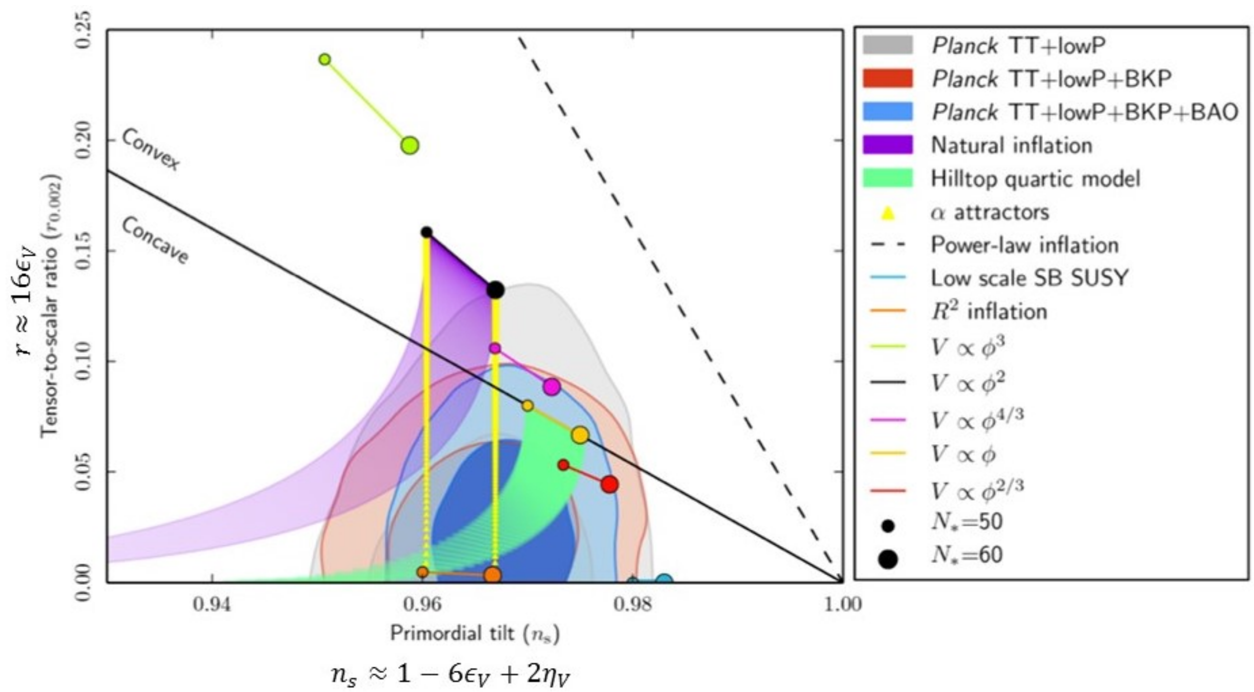

Figure 1: Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets, compared to theoretical predictions of selected inflationary models.

The most stringent bounds are the dark blue region (68% confidence level (CL)) and the light blue region (95% CL). Usually a model is not considered to be excluded, if it is within the 2σ (i.e. 95% CL) region. Once it is outside this region it is somewhat or very unlikely depending on how far outside it is.

Theoretical models are shown in this plots as dots connected by a line or as a band. The two dots that are shown for particular models, like for example the two black dots for $V \propto \phi^2$ correspond to $N_e = 50$ and $N_e = 60$ and the line connecting the dots correspond to N_e values between 50 and 60. The reason that this range is shown is that we do not know for sure the exact number of e-folds N_e that we can observe.

In the sense discussed above, the most simple model of inflation, the $m^2\phi^2$ model we discussed last time, just became very unlikely and other models seem favored by the data. For example, as you have seen in the homework, models with $V \propto \phi^p$ are more favored for $p < 2$. For the model of natural inflation that we discussed last time we see a purple band. The reason for this is that this model has the free parameter f and depending on the value of this parameter we get different predictions. So f in this model is constraint by the data to take certain values. We also see that the purple band is connected to the $m^2\phi^2$ model that we discussed last time as limiting case of natural inflation.

Note that the parameter r determines directly the energy scale during inflation via the equation $V_{inf} \approx 2 \times 10^{16} GeV \left(\frac{r}{.1}\right)^{\frac{1}{4}}$. While experiments so far have only placed an upper bound $r < .07$ (95% CL), any future detection of a non-zero r would tells us directly the energy scale at which inflation took place.

The figure above shows a few more models of inflation that are just a very small selection of the actual models that have been proposed. Hopefully in the future the ever improving observations will narrow the parameter space down to a single model.

2 Beyond slow-roll single field inflation

As you can already guess from the above plot, there are a lot of single field slow-roll models since we can tune the parameters in many scalar potentials such that they contain sufficiently flat regions. In addition to these single field slow-roll models there are also multifield models in which we don't just have a single scalar field but many. These models are obviously much more complicated and make a variety of interesting predictions. None of these have been observed so far so that multifield models are constrained (but by no means excluded).

In addition to these slow-roll models in which a scalar field is rolling very slowly, it is also possible to get a period of accelerated expansion from a very fast rolling scalar field. This seemingly counterintuitive statement follows from the fact that we can't neglect higher order corrections to the scalar field's kinetic term, once the field is fast rolling, i.e. terms like $(\partial_\mu \phi \partial^\mu \phi)^n$ for $n > 1$ become important. If these higher order corrections take a specific form, then the fast rolling scalar field leads to a period of inflation. This class of models is usually called K-inflation, where K stands for 'kinetic'.

3 Reheating

Inflation solves several problems in our universe but it also drastically modifies the evolution of our early universe. In particular, the rapid expansion during inflation leads to an essentially empty universe since the contributions to the energy density from matter, radiation and curvature scale with a negative power of the scale factor $a(t)$. Since $a(t)$ grows by a huge factor, essentially the entire energy density of the universe is given by the contribution of the scalar field:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{3}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{4}$$

At the end of inflation one of the slow-roll parameters becomes of order 1, which is equivalent to the kinetic term $\dot{\phi}^2$ becoming important. If the scalar field would oscillate around the minimum at which $V(\phi) \approx 0$, then we would have $\rho_\phi \approx P_\phi$, i.e. a phase with $w = 1$. In that case it follows from equation (5) in the lecture 2 notes that $\rho_\phi \propto a(t)^{-3(1+w)} = a(t)^{-6}$. The universe at this point keeps expanding since the first Friedmann equation becomes

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{\dot{\phi}^2}{6M_P^2} > 0\,. \tag{5}$$

This would mean that the energy density of the universe would go to zero and we would be left with a completely empty universe. Likewise, if the inflaton oscillates around the minimum of the potential and the Hubble friction is negligible (because H becomes small at the end of inflation), then the kinetic energy and the potential energy are on average the same $\frac{1}{2}\langle\dot{\phi}^2\rangle = \langle V(\phi)\rangle$ and the inflaton behaves like pressure-less matter $\langle P_\phi\rangle = 0$. In this case we have $\rho_\phi \propto a(t)^{-3}$. So what we need is a way to transfer the kinetic energy of the scalar field into the standard model degrees of freedom so that our hot big bang scenario can take place.

The period during which this energy transfer is happening is called reheating, since the universe during inflation has essentially zero temperature [1] and then the universe gets reheated. This reheating can be accomplished by coupling the inflaton scalar field ϕ to other fields like for example the fermions ψ in the standard model via $\phi\bar{\psi}\psi$ Yukawa couplings in the action. These then lead to an extra term in the equation of motion of the inflaton

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi\dot{\phi} + V'(\phi) = 0\,. \tag{6}$$

The additional friction term Γ_ϕ is proportional to the decay width of the inflaton that in turn is determined by the prefactor of the above Yukawa coupling. During inflation we need $H \gg \Gamma_\phi$ so that the above term does not spoil inflation. At the end of inflation H decreases and then the term Γ_ϕ will become important and the energy transfer from the inflaton to the standard model particles will take place.

This leads then to a reheating of the universe. Note that the spatial inhomogeneities of the inflaton field $\nabla^2\phi/a(t)^2$ have been sufficiently suppressed during inflation to avoid the horizon problem and the reheating will naturally lead to a homogeneous temperature distribution. If the inflaton only couples to some of the standard model particles, then the ample interactions that take place will lead to the required initial conditions in which all standard model particles are in thermal equilibrium.

During inflation the scalar field has essentially only potential energy that then gets transferred into kinetic energy towards the end of inflation. This kinetic energy then gets in turn transferred into the standard model particles as soon as the Hubble parameter has decreased sufficiently at the end of inflation so that $H \lesssim \Gamma_\phi$. This means that the maximum reheating temperature is set by the energy scale during inflation $T_{reh,max} \sim V_{inf}^{\frac{1}{4}}$. This energy scale during inflation is currently constrained by observation to be smaller than roughly the GUT scale, which is $10^{16}GeV$. This means in turn that the reheating temperature cannot be larger than the GUT scale so that after inflation there is no problem with relics that might appear during the breaking of the GUT gauge group to the standard model gauge groups.

The lower end for the reheating temperature and therefore also for the energy density during inflation is essentially only constrained by our observation of the correct amount of light elements like Helium and Hydrogen. As we discussed, this can be correctly explained by the nucleosynthesis which starts around a few MeV. So although we in principle understand physics up to much larger energy scales of $1TeV$, it is possible that at the end of inflation the universe only gets reheated to a temperature of a few MeV and then the thermal evolution we discussed takes place. This means that there is a huge energy range $.001\,GeV \lesssim V_{inf}^{\frac{1}{4}} \lesssim 10^{16}\,GeV$ during which inflation could have happened.

Note that the reheating temperature T_{reh} does not have to be equal to the energy density at the end of inflation but is only bounded by it from above. It is possible that the reheating

[1] Based on the stretching of the wavelength λ of photons we argued that $T \propto a(t)^{-1}$, so that the temperature will decrease very quickly during inflation and is negligible compared to the inflationary energy $V_{inf}^{\frac{1}{4}}$ which is approximately constant during inflation.

temperature is lower than the energy density during inflation. However, the exact relation requires a specific model of inflation with a specific reheating mechanism.

4 The inhomogeneous universe

So far in the course we have restricted ourselves to a completely homogeneous and isotropic universe. While this is a very good approximation for our universe on large scales, it is certainly not true on smaller scales like the size of galaxies. How can we explain these observed inhomogeneities and thereby our very own existence?

A natural thought might be that the unknown initial conditions of our universe were not perfectly homogeneous and through time gravitational clumping leads to structure formation. A problem with this is that a sufficiently long period of inflation erases any initial inhomogeneities. So it seems that a period of inflation seems to make the existence of inhomogeneities and structure formation more difficult. However, this is not the case. The reason is Heisenberg's uncertainty principle that applies to quantum mechanics and also to quantum field theory. Since our universe is governed by a quantum theory we can not just treat the inflaton field as a classical scalar field but we have to take quantum mechanics into account. Heisenberg's uncertainty principle says that we cannot exactly determine the value of the field ϕ and its conjugate momentum. This means that we should allow for small fluctuations in ϕ:

$$\phi = \bar{\phi}(t) + \delta\phi(t, \vec{x})\,. \tag{7}$$

Here we have decomposed the field ϕ into a background field $\bar{\phi}(t)$ that is homogeneous and satisfies the classical equations of motion and a small fluctuation that depends on space and time, i.e. that is not homogeneous. Likewise we can perturb the metric $g_{\mu\nu}$ and the energy density and pressure. As long as these perturbations are small, we can derive their equations of motion by linearizing the equations of motion. We can then study these linear equations of motion and quantize the fluctuations. We can track the evolution of these fluctuation and derive their features. While this is not too complicated it is somewhat technical and a complete treatment would take several more lectures so we will skip the details and focus on the results.

The quantum fluctuations of the inflaton field mean that the position of the inflaton field is different at different points in space (since $\delta\phi$ depends on $\vec{x}$). This is sketched in figure 2.

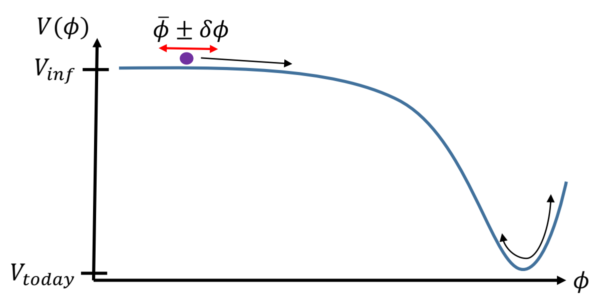

Figure 2: The position ϕ of the scalar field changes in space and time due to quantum fluctuations $\delta\phi(t, \vec{x})$.

What this means, and what one finds by solving the equations for the perturbations, is that inflation ends at slightly different times at different points in space. This then leads to slightly more or less dense regions at the end of inflation. We can explicitly calculate the evolution of the fluctuations during and after inflation as long as they are small and perturbation theory is applicable. What one finds is that small quantum fluctuations are getting stretched and amplified during inflation. At the end of inflation the perturbations grow slowly during the radiation dominated epoch and then more rapidly during the matter dominated epoch. The gravitational attraction will eventually make the perturbations so large that structures like stars and galaxies will form. At this point the linear perturbation theory has broken down. However, the density fluctuations lead to small temperature fluctuations in the cosmic microwave background that we can observe and theoretically calculate using linearized perturbation theory. The latest experimental observation of these temperature fluctuations from the Planck satellite is shown in figure 3.

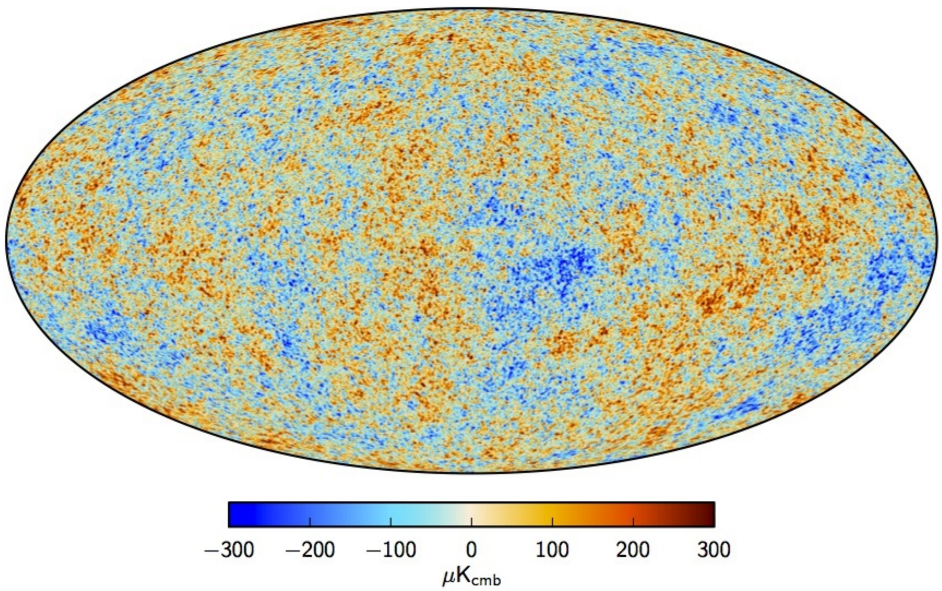

Figure 3: Very small temperature fluctuations in the CMB as observed by the Planck satellite.

The CMB photons that we measure come from a huge sphere that surrounds us and similar to a map of the surface of the earth we can project this sphere on a 2-dimensional surface, which is shown in the figure. The temperature $T = 2.725K$ is very homogeneous and the root mean square variations are only $18\mu K$ so that the variations in the temperature are roughly $\delta T/T \sim 10^{-5}$. These small fluctuations look like random noise and that is exactly what they are: random quantum noise. This noise nevertheless contains a tremendous amount of information. For initial quantum noise that evolves during the radiation and matter dominated era until the release of the CMB 380,000 years after the big bang, one can derive very precisely the expected 2-point function for the temperature fluctuations. This 2-point function depends only on the angle φ between the two points on the sky and we can expand it therefore in terms of Legendre Polynomials $P_l(\cos(\varphi))$. The resulting theoretical curve together with the data is shown in figure 4. We see that the data and theory beautifully agree with each other and that the random quantum noise indeed contains a structure that is not visible in figure 3.

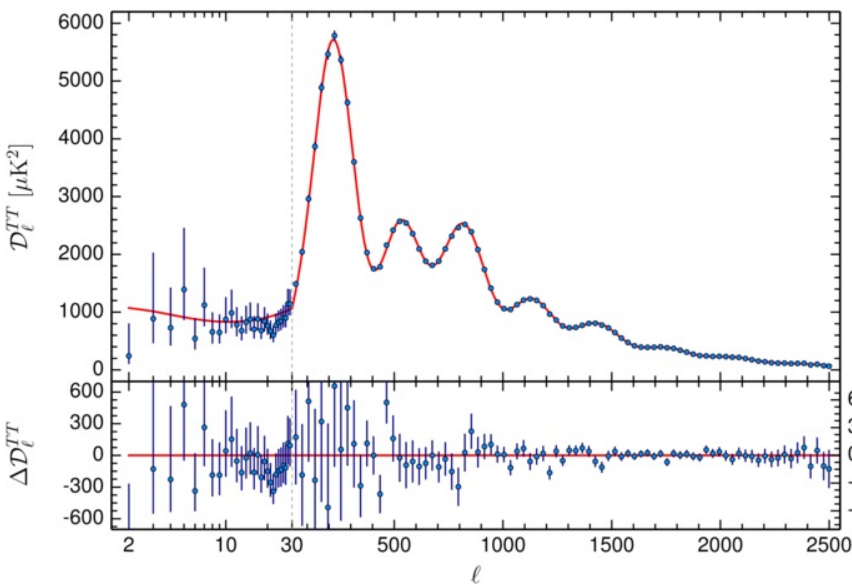

Figure 4: The 2-point function of the temperature fluctuations as a function of the multipole l. The lower part is zoomed in and shows the tiny differences between the red theory curve and the data with 1σ error bars.

From the measured fluctuations we can determine the ratio of the inflaton potential $V(\phi)$ and the slow-roll parameter ϵ_ϕ 50 to 60 e-folds before the end of inflation. Concretely from the detailed perturbation theory calculation one finds for slow-roll models that

$$\frac{1}{24\pi^2}\frac{V(\phi)}{M_P^4}\frac{1}{\epsilon_V} \approx 2.2 \times 10^{-10}\,. \tag{8}$$

This equation allows us to fix a parameter in our inflationary model. For example for $V(\phi) = \frac{1}{2}m^2\phi^2$ we calculated last time for $N_e = 60$ that $\phi_i = 15.6M_P$ and $\epsilon_V = \frac{1}{121}$, so that

$$\frac{1}{24\pi^2}\frac{V(\phi)}{M_P^4}\frac{1}{\epsilon_V} = \frac{1}{48\pi^2}\frac{m^2(15.6M_P)^2}{M_P^4}121 \approx 62\frac{m^2}{M_P^2} \approx 2.2 \times 10^{-10}\,. \tag{9}$$

Recalling that $M_P \approx 2.4 \times 10^{18} GeV$ we find

$$m \approx 2.4\sqrt{\frac{2.2}{62}}10^{13}GeV \approx 4.5 \times 10^{12}GeV\,. \tag{10}$$

So we have found that the inflaton in this model is much heavier than the all the particles in the standard model that have masses up to only $172\,GeV$.

In addition to the temperature fluctuations in the CMB one can in principle also measure the polarization of the CMB photons. This would provides us with additional information about the perturbations that were generated during inflation. In particular the fluctuations in the space-time metric that are gravitational waves and that are generated during inflation give rise to particular patterns in the polarization of the CMB photons, as is shown in figure 5.

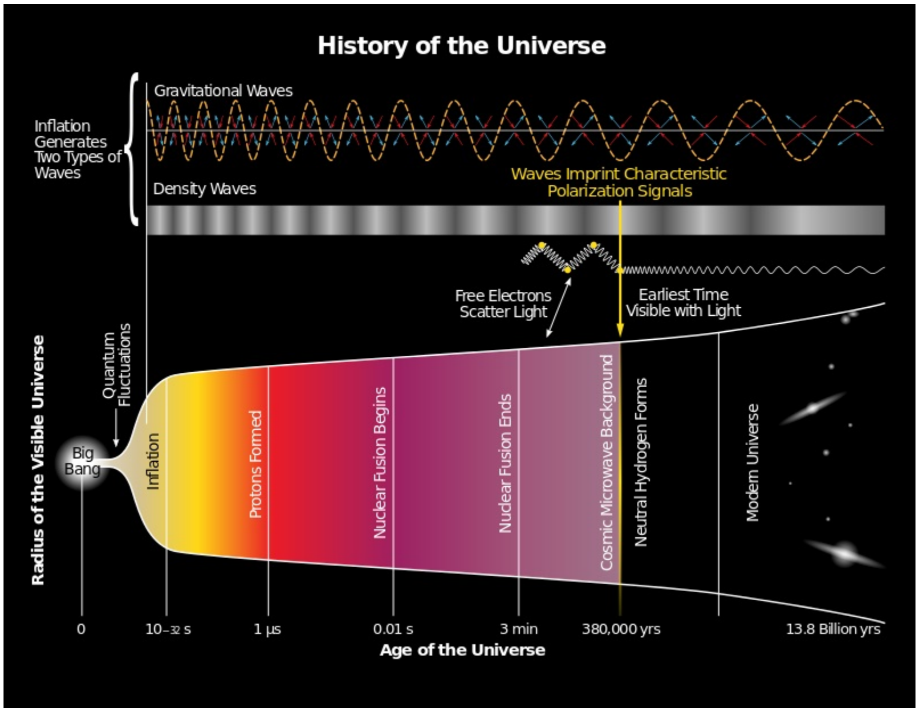

Figure 5: During inflation density and metric perturbations are generated. They can indirectly be measured by observing the temperature fluctuations and the polarization of the CMB photons.

This is particularly interesting because contrary to the density/temperature fluctuations that are enhanced by a factor $1/\epsilon_V$ (see equation 8), the so called tensor perturbations are not enhanced. Thus they are much smaller and harder to measure but at the same time any detection would tells us the actual energy scale during inflation (since the unknown enhancement from $1/\epsilon_V$ is absent). However, so far we have not been able to detect this particular type of polarization in the CMB photons so that we only have an upper bound on the so called scalar-to-tensor ratio $r < .07$ (95% CL). The energy scale during inflation is then determine through this parameter r as

$$V_{inf}^{\frac{1}{4}} \approx 2 \left(\frac{r}{.1}\right)^{\frac{1}{4}} 10^{16} GeV\,. \tag{11}$$

Future experiments can detect r or lower the bound potentially up to $r \gtrsim 10^{-4}$ so that any future detection of r would corresponds to a very high scale of inflaton. In particular this scale would be roughly 12 orders of magnitude larger than what can be tested in particle accelerators! This energy scale would also be very close to the Planck scale which might allow us to get insights into the theory of quantum gravity that governs our universe. So the future of theoretical and observational cosmology holds the promise of great discoveries!